POL 213: Problem Set 1
Fun With Distributions

Please answer the following questions. With your answer, please turn in your R log-file. I am being intentionally vague on R script in this assignment in hopes that you all can figure some of this out. It will be tough and I can answer questions as need be.

1. Normal Distribution: The likelihood function for the normal distribution when $\sigma = 1$ is given by:

$$\begin{aligned} \mathcal{L}(\theta \mid y) &= \prod_{i=1}^{N} f(y_i \mid \mu, \sigma = 1) \\ &= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(y_i - \mu)^2}{2}\right) \end{aligned} \tag{1}$$

In R, write a function that finds a value, μ, that maximizes the log-likelihood of **y**, where **y** is a vector of ten observations drawn randomly from a standard normal distribution. Generate 20 possible values for μ and plot the resulting log-likelihoods with respect to the μ_i. Label the graph axes appropriately. For what value of μ is the log-likelihood maximized? To verify your answer, solve the first derivative. For this case, the first derivative is given by

$$\frac{\partial}{\partial \mu}\left[-\sum_{i=1}^{N}(y_i - \mu)^2\right] = 2\sum_{i=1}^{N} y_i - 2N\mu. \tag{2}$$

What value of μ satisfies the first order condition? In ordinary statistics, what does μ correspond to? (20 points)

2. Poisson Distribution: Suppose the likelihood function for the Poisson distribution is given by

$$\mathcal{L}(\theta \mid y) = \frac{e^{-N\lambda}\lambda^{\sum y_i}}{\prod y_i!}. \tag{3}$$

Note that the Poisson distribution is commonly used to describe the DGP for variables of counts. For this problem, create a vector of 10 positive integers (0 is also ok). Keep the integers within the range [0,5]. First do a grid search and find the value of λ that maximizes the function. Plot the likelihood with respect to lambda. After inspecting this plot, redo your grid search using a finer mesh. To help get you started, note that candidate values for λ can be generated by:

```
lambda <- seq(0,5, .01)
```

After this, you need to write the function given in equation 3 and plot it. The fine mesh grid can be constructed using the code fragment from above. Describe the location of the maximum in both plots and describe the differences you see between the first and second grid searches. (20 points)

3. Using code given in lecture slides, estimate a logit model and show, using the Hessian, what the standard errors of the regression coefficients should be. To do this, generate two vectors of data where the first vector is a binary sequence (1s, 0s) having 100 elements and the second vector is some random variable x, defined however you want to define (i.e. it can be random if you want).(20 points)