Count Models

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Today: Event Count Models
Topics

- Poisson Distribution
- Different “Flavors” of Event Count Models
Motivation

- Nature of the Response Variable, $y$
- Non-negative integer, $y = 0, 1, 2, \ldots, k$
- Examples:
  - Coups d’Etat
  - Presidential Vetoes
  - Cabinet Terminations
  - Tornado Deaths
- Events occur in an interval of time.
- Frequently the case that $Pr(Event)$ is small, but $n$ time periods is large.
- Usually assume the observation period is fixed (i.e. it doesn’t vary).
Issues

- The response variable is “absolutely continuous.”
- Normal distribution does a poor job of describing DGP.
- Normal models, like OLS, may return negative predicted values.
- ...while having similar kinds of problems we saw wrt categorical variables.
The Poisson

- Useful to consider the Poisson distribution.
- Poisson distribution describes the probability of events occurring within \( t, t + 1 \).
- Probability function is given by:
  \[
  \Pr(Y_i = y) = \frac{\exp(-\lambda_i)\lambda_i^y}{y!}
  \]
- \( \lambda \) gives us the “expected number of counts.”
- The PDF is asymmetric for small \( \lambda \) (Why?)
Simulation: 1000 draws from a Poisson, lambda=1, 5, 20, 100

```r
samplesize <- 1000
lambda_1 <- sample(rpois(samplesize, 1))
hist(lambda_1, br=c(0,1,2,3,4,5,6,7,8,9,10), col="blue1")

lambda_5 <- sample(rpois(samplesize, 5))
hist(lambda_5, br=c(0:20), col="blue1")

lambda_20 <- sample(rpois(samplesize, 20))
hist(lambda_20, br=c(0:50), col="blue1")

lambda_100 <- sample(rpois(samplesize, 100))
hist(lambda_100, br=c(0:150), col="blue1")
```
$\lambda = 1$

Histogram of $\lambda_1$
$\lambda = 5$

Histogram of \( \lambda_5 \)
Today: Event Count Models

$$\lambda = 20$$
$$\lambda = 100$$

Histogram of \(\lambda_{100}\)
Some Properties of Poisson

- This is a single parameter distribution.
- Implies: $E(Y) = \lambda = \text{var}(\lambda)$
- Thus, the variance is increasing with the mean.
- We may be interested in modeling expected counts as a function of covariates.
- This is the Poisson Model.
The Poisson Model

- Expectation as a function of covariates:
  \[ E[y_i \mid x] = \lambda = \exp(\beta' x_i) \]

- As probability:
  \[ \Pr(Y_i = y) = \frac{e^{-e^{\beta' x_i}} (e^{\beta' x_i})^y}{y_i!} \]

- As log-linear model (i.e., linear model for \( \log E[y_i \mid x] \)):
  \[ \log E[y_i \mid x] = \beta' x_i \]

- Cameron and Trivedi refer to the first two functions as the Poisson regression model.

- Likelihood:
  \[ \log L = \sum_{i=1}^{n} \left\{ -\exp(\beta' x_i) + y_i \beta' x_i - \log y_i! \right\} \]
Interest often centers on “expected counts” given some covariate:

\[ E[y_i \mid x] = \lambda = \exp(\beta' x_i) \]

Or marginal effects:

\[ \frac{\partial E[y_i \mid x_i]}{\partial x_i} = \exp(\beta' x_i) \beta \]
Illustration

- Use Fred Boehmke’s data from 2005 *PRQ*.
- Response variable: count of ballot initiatives.
- *Stata* first; *R* second.
. poisson ballot citizen econ statinit consinit sigs unlimit circdays singlesu distribu primaryo repgov repleg i 
> deology legprof time minordiv deficit, cluster(stateno)

Iteration 0:  
log pseudolikelihood = -508.68756

Iteration 1:  
log pseudolikelihood = -504.7026

Iteration 2:  
log pseudolikelihood = -504.6913

Iteration 3:  
log pseudolikelihood = -504.6913

Poisson regression
Number of obs = 277
Wald chi2(17) = 889.33
Log pseudolikelihood = -504.6913
Prob > chi2 = 0.0000
(Std. Err. adjusted for 23 clusters in stateno)

| ballot          | Coef.   | Std. Err. | z     | P>|z| | [95% Conf. Interval] |
|-----------------|---------|-----------|-------|-----|-----------------------|
| citizen         | .2706867| .0658299  | 4.11  | 0.000 | .1416624 - .399711    |
| econ            | -.1365263| .0462521 | -2.95 | 0.003 | -.2271788 - .0458738  |
| statinit        | 3.234443 | .6568364 | 4.92  | 0.000 | 1.947068 4.521819    |
| consinit        | .6646655 | .1363439 | 4.87  | 0.000 | .3974363 .9318947    |
| sigs            | -.1652336| .0299768 | -5.51 | 0.000 | -.2239731 - .1064802 |
| unlimit         | .4954682 | .180718  | 2.74  | 0.006 | .1412674 .8496689    |
| circdays        | .0785881 | .0360028 | 2.18  | 0.029 | .008024  .1491523    |
| singlesubt      | .4259296 | .1301254 | 3.27  | 0.001 | .1708884 .6809707    |
| distribution    | -.8871728| .1188417 | -7.47 | 0.000 | -1.120098 -.6542474  |
| primaryoffr     | -.2958509| .1292109 | -2.29 | 0.022 | -.5490997 -.0426021  |
| repgov          | -.1339101| .1015792 | -1.32 | 0.187 | -.3330018 .0651816   |
| repleg          | -.2437057| .1255485 | -1.94 | 0.052 | -.4897762 .0023649   |
| ideology        | 3.140647 | .6233684 | 5.04  | 0.000 | 1.918867 4.362426    |
| legprof         | -.6318353| .4315538 | -1.46 | 0.143 | -1.477665 .2139946   |
| time            | .0122646 | .0103938 | 1.18  | 0.238 | -.008107 .0326362    |
| minordiv        | 2.122436 | 1.019348 | 2.08  | 0.037 | .1243744 4.120497    |
| deficit         | -1.923051| 1.178353 | -1.63 | 0.103 | -4.232581 .3864794   |
| _cons           | -1.48057 | .6620717 | -2.24 | 0.025 | -2.778207 -1.1829339 |
Call:
glm(formula = ballot ~ citizen + econ + statinit + consinit +
sigs + unlimit + circdays + singlesubject + distribution +
primaryoffyear + repgov + repleg + ideology + legprof + time +
minordiv + deficit, family = poisson())

Deviance Residuals:
Min 1Q Median 3Q Max
-2.5581 -1.1366 -0.2113 0.6015 3.5198

Coefficients:

Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.48057 0.44301 -3.342 0.000832 ***
citizen 0.27069 0.08063 3.357 0.000788 ***
econ -0.13653 0.04891 -2.792 0.005245 **
statinit 3.23444 0.40877 7.913 2.52e-15 ***
consinit 0.66466 0.13958 4.762 1.92e-06 ***
sigs -0.16523 0.02431 -6.796 1.08e-11 ***
unlimit 0.49547 0.17567 2.820 0.004797 **
circdays 0.07859 0.03494 2.250 0.024479 *
singlesubject 0.42593 0.11325 3.761 0.000169 ***
distribution -0.88717 0.09667 -9.177 < 2e-16 ***
primaryoffyear -0.29585 0.12358 -2.394 0.016663 *
repgov -0.13391 0.08047 -1.664 0.096099 .
repleg -0.24371 0.09612 -2.535 0.011233 *
ideology 3.14065 0.79799 3.936 8.29e-05 ***
legprof -0.63184 0.32019 -1.973 0.048462 *
time 0.01226 0.00711 1.725 0.084516 .
minordiv 2.12244 0.64540 3.289 0.001007 **
deficit -1.92305 0.81840 -2.350 0.018785 *

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 928.24  on 276  degrees of freedom
Residual deviance: 405.42 on 259  degrees of freedom
(27 observations deleted due to missingness)
AIC: 1045.4
Number of Fisher Scoring iterations: 6

Call:
\[
\text{vglm(formula = ballot \sim citizen + econ + statinit + consinit + \\
\quad \text{sigs} + \text{unlimit} + \text{circdays} + \text{singlesubject} + \text{distribution} + \\
\quad \text{primaryoffyear} + \text{repgov} + \text{repleg} + \text{ideology} + \text{legprof} + \text{time} + \\
\quad \text{minordiv} + \text{deficit}, \text{family} = \text{poissonff})}
\]

Pearson Residuals:
\[
\begin{array}{cccccc}
\text{Min} & 1Q & \text{Median} & 3Q & \text{Max} \\
\text{log(mu)} & -2.0047 & -0.90063 & -0.20443 & 0.68061 & 6.2293 \\
\end{array}
\]

Coefficients:
\[
\begin{array}{cccccc}
\text{(Intercept)} & -1.480570 & 0.4429487 & -3.3425 & \\
\text{citizen} & 0.270687 & 0.0806334 & 3.3570 & \\
\text{econ} & -0.136526 & 0.0489062 & -2.7916 & \\
\text{statinit} & 3.234443 & 0.4087049 & 7.9139 & \\
\text{consinit} & 0.664665 & 0.1395794 & 4.7619 & \\
\text{sigs} & -0.165234 & 0.0243137 & -6.7959 & \\
\text{unlimit} & 0.495468 & 0.1756689 & 2.8205 & \\
\text{circdays} & 0.078588 & 0.0349338 & 2.2496 & \\
\text{singlesubject} & 0.425930 & 0.1132493 & 3.7610 & \\
\text{distribution} & -0.887173 & 0.0966693 & -9.1774 & \\
\text{primaryoffyear} & -0.295851 & 0.1235732 & -2.3941 & \\
\text{repgov} & -0.133910 & 0.0804709 & -1.6641 & \\
\text{repleg} & -0.243706 & 0.0961210 & -2.5354 & \\
\text{ideology} & 3.140647 & 0.7979709 & 3.9358 & \\
\text{legprof} & -0.631835 & 0.3201900 & -1.9733 & \\
\text{time} & 0.012265 & 0.0071095 & 1.7251 & \\
\text{minordiv} & 2.122436 & 0.6453928 & 3.2886 & \\
\text{deficit} & -1.923051 & 0.8183928 & -2.3498 & \\
\end{array}
\]

Number of linear predictors: 1
Name of linear predictor: log(mu)
(Default) Dispersion Parameter for poissonff family:  1
Residual Deviance:  405.4232 on 259 degrees of freedom
Log-likelihood:  -504.6913 on 259 degrees of freedom
Number of Iterations:  5
Illustration

- Stata, use `poisson`
- R, I used `glm` and `vglm`
- Happily, all yield identical results.
- In R, I couldn’t figure out how to suppress the intercept (which F.B. does in his article). It makes no difference though (just remove `statecons` from model).
- Interpretation?
  - Would want to compute odds-like ratios: $\exp(\beta x)$
  - Expected counts for covariate profiles
  - Marginal effects.
Recall that in Poisson, mean = variance.

Poisson model presumes Poisson process:
The probability of an event occurring within some period (usually measured temporally) is constant and independent of all previous events.

If events tend to be “lumpy” (clustering) or non-independent, dispersion may exist.

Convenient to think of “over” vs. “under” dispersion.
Dispersion

- Overdispersion is common. Events are more prone for some sample elements compared to others. Event amass at different periods (probability is not constant). The occurrence of past events effect probability of future events.

- Underdispersion is apparently uncommon. Occurrence of past events decreases chances for future events. Evidently not widely found in social science literature.
Dispersion

- Conceptually:

\[ \sigma^2 = 1, \quad E(y_i) = \text{var}(y_i) \]
\[ \sigma^2 > 1, \quad E(y_i) < \text{var}(y_i) \]
\[ \sigma^2 < 1, \quad E(y_i) > \text{var}(y_i) \]

- The first is dispersion under the Poisson; the second is overdispersion; the third is underdispersion.

- Dispersion relates to efficiency of the estimator.

- Specifically, estimated parameters are consistent, but s.e. are inefficient and inconsistent.

- Leads to consideration of alternative modeling strategies.
Over Dispersion

- Focus will be on over dispersion.
- Are there tests for overdispersion?
- Cameron and Trivedi (1998) propose a regression-based test:

\[
\frac{(y_i - \hat{\lambda}_i)^2 - y_i}{\hat{\lambda}_i} = \alpha \hat{\lambda}_i + \epsilon_i
\]

\[
\frac{(y_i - \hat{\lambda}_i)^2 - y_i}{\hat{\lambda}_i} = \delta + \epsilon_i
\]

where $\hat{\lambda}_i = \exp(\beta' x_i)$. If Poisson dispersion holds, the mean function should have no effect on the variance function.

- Model-based test: negative binomial vs. Poisson.
What follows is adapted from Long (1997).

The motivation:
\[ \tilde{\lambda}_i = \exp(\beta' x_i + \epsilon_i) \]

The problem:
\[ \tilde{\lambda}_i = \exp(\beta' x_i) \exp(\epsilon_i) \]
\[ = \lambda_i \exp(\epsilon_i) \]
\[ = \lambda_i \delta_i \]

where \( \delta_i = \exp(\epsilon_i) \).
We need structure on $\delta_i$ in order to proceed.

Convenient to assume $E(\delta_i) = 1$.

For our purposes, this yields a nice result:

$$E(\tilde{\lambda}_i) = E(\lambda_i \delta_i)$$
$$= \lambda_i E(\delta_i)$$
$$= \lambda_i.$$

And in probabilities:

$$\Pr(Y_i = y \mid x, \delta_i) = \frac{\exp(-\tilde{\lambda}_i) \tilde{\lambda}_i^y}{y_i!}$$
$$= \frac{\exp(-\lambda_i \delta_i) (\lambda_i \delta_i)^y}{y_i!}$$

(1)

We now have a Poisson process.
Negative Binomial Model

- The problem is, $\delta_i$ is unknown and so we cannot directly estimate the previous function.
- What do we do?

$$\Pr(Y_i \mid x_i) = \int_{0}^{\infty} \left[ \Pr(y_i \mid x_i\delta_i) \times g(\delta_i) \right] d\delta_i$$  \hspace{1cm} (2)

- What we’re doing here is integrating over values of $\delta_i$. In a sense, we’re averaging over the probability of each $\delta_i$.

- From Long (1997, p. 231), suppose $\delta_i$ has two values, $d_1$ and $d_2$:

$$\Pr(Y_i \mid x_i) = \left[ \Pr(y_i \mid x_i\delta_i = d_1) \times \Pr(\delta_i = d_1) \right] + \left[ \Pr(y_i \mid x_i\delta_i = d_2) \times \Pr(\delta_i = d_2) \right]$$
In order to do all this, we need to specify a PDF for $\delta_i$.

A convenient and commonly used distribution here is the gamma.

$$
g(\delta_i) = \frac{\nu_i^{\nu_i}}{\Gamma(\nu_i)} \delta_i^{\nu_i-1} \exp(-\delta_i \nu_i)
$$

$\Gamma$ is the gamma function.

Under the gamma, $E(\delta_i) = 1$ (which is what we wanted) and $\text{var}(\delta_i) = 1/\nu_i$. 

Solve equation 2 using equation 1 for $\Pr(Y_i = y \mid x, \delta_i)$ and equation 3 for $g(\delta_i)$ and you obtain the negative binomial model:

$$
\Pr(Y_i \mid x_i) = \frac{\Gamma(y_i + \nu_i)}{y_i!\Gamma\nu_i} \left( \frac{nu_i}{\nu_i + \lambda_i} \right)^{\nu_i} \left( \frac{\lambda_i}{\nu_i + \lambda_i} \right)^{y_i}
$$

Under this, $E(y_i \mid x_i) = \exp(\beta'x_i)$ (which is good).

The mean function is same as Poisson, but the variance is different.

$$
\text{var}(y_i \mid x_i) = \lambda \left( 1 + \frac{\lambda_i}{\nu_i} \right)
$$
Negative Binomial Model

- One thing left to do: put an identifying restriction on \( \nu_i \).

\[
\nu_i = \alpha^{-1}
\]

is standard.

- By removing the subscript \( i \), we've turned the variance into a constant.

- This gives a variance function of:

\[
\text{var}(y_i \mid x_i) = \lambda_i + \alpha \lambda_i^2
\]

(Proof given in Cameron and Trivedi, Chapter 3)

- \( \alpha \) is a dispersion parameter. As it approaches 0, the Poisson model is obtained.
Negative Binomial Model

- Likelihood

\[ L = \prod_{i=1}^{N} \frac{\Gamma(y_i + \alpha^{-1})}{y_i! \Gamma(\alpha^{-1})} \left( \frac{\alpha^{-1}}{\alpha^{-1} + \lambda_i} \right)^{\alpha^{-1}} \left( \frac{\lambda_i}{\alpha^{-1} + \lambda_i} \right)^{y_i} \]

- Note: often \( \log(\alpha) \) is used in estimation; report \( \alpha \), however.

- Interpretation: covariates have same interpretation as Poisson because mean function is the same. Standard interpretative tricks apply.

- Illustration using Fred Boehmke’s data.
Negbinomial regression
Number of obs = 277
Dispersion = mean
Wald chi2(17) = 635.65
Log pseudolikelihood = -498.53791 Prob > chi2 = 0.0000

(Std. Err. adjusted for 23 clusters in stateno)

+-----------------------------+-----------------------------+-----------------------------+-----------------------------+-----------------------------+-----------------------------+
|                | Robust                |                |                |                |                |
|                | Coef.                  | Std. Err.      | z               | P>|z|            | [95% Conf. Interval]        |
|-----------------------------+-----------------------------+-----------------------------+-----------------------------+-----------------------------+-----------------------------|
| ballot | citizen | .2965608 | .071057 | 4.17 | 0.000 | .1572917 | .4358299 |
| economic | econ | -.1559814 | .048059 | -3.25 | 0.001 | -.2501753 | -.0617875 |
| statinit | statinit | 3.309358 | .6821106 | 4.85 | 0.000 | 1.972446 | 4.64627 |
| consinit | consinit | .7045804 | .1454361 | 4.84 | 0.000 | .4195309 | .9896299 |
| sigs | sigs | -.1681165 | .0294342 | -5.71 | 0.000 | -.2258065 | -.1104265 |
| unlimit | unlimit | .4657434 | .1805621 | 2.58 | 0.010 | .1118481 | .8196386 |
| circdays | circdays | .0847279 | .035506 | 2.39 | 0.017 | .0151374 | .1543184 |
| singlesubject | singlesubject | .421773 | .1303808 | 3.23 | 0.001 | .1662313 | .6773146 |
| distribution | distribution | -.8714067 | .116634 | -7.47 | 0.000 | -1.100005 | -.6428081 |
| primaryofficer | primaryofficer | -.2499797 | .1272447 | -1.96 | 0.049 | -.4993746 | -.0005847 |
| repgov | repgov | -.1231686 | .1046831 | -1.18 | 0.239 | -.3283437 | .0820064 |
| repleg | repleg | -.2718297 | .1269838 | -2.14 | 0.032 | -.5207134 | -.0229461 |
| ideology | ideology | 3.270144 | .633435 | 5.16 | 0.000 | 2.028634 | 4.511654 |
| legprof | legprof | -.7854763 | .4351239 | -1.81 | 0.071 | -.1638303 | .0673509 |
| time | time | .0108917 | .0097875 | 1.11 | 0.266 | -.0082914 | .0300749 |
| minordiv | minordiv | 2.235202 | .9640336 | 2.32 | 0.020 | .3457311 | 4.124673 |
| deficit | deficit | -1.961033 | 1.146748 | -1.71 | 0.087 | -.4208619 | .2865523 |
| _cons | _cons | -1.519194 | .6931931 | -2.19 | 0.028 | -2.877827 | -.1605602 |
| /lnalpha | /lnalpha | -2.220836 | .439864 | -3.082954 | 1.358719 |
| alpha | alpha | .1085183 | .0477333 | 0.0458237 | 0.2569899 |
Call:
glm.nb(formula = ballot ~ citizen + (bunch of vars), init.theta = 9.21509710247016, 
link = log)

Deviance Residuals:
          Min       1Q     Median       3Q      Max
-2.4223  -1.0039  -0.2209   0.5445   2.8634

Coefficients: (1 not defined because of singularities)
                     Estimate Std. Error    z value Pr(>|z|)
(Intercept)       -1.519194   0.488739   -3.108  0.001881 **
citizen            0.296561   0.095508    3.105  0.001902 **
econ               -0.155981   0.058512   -2.666  0.007680 **
statinit           -0.168117   0.027479   -6.118  9.48e-10 ***
consinit           -0.168117   0.027479   -6.118  9.48e-10 ***
sigs               -0.168117   0.027479   -6.118  9.48e-10 ***
unlimit            0.465744   0.200723    2.320  0.020323 *
circdays           0.084728   0.038738    2.187  0.028728 *
singlesubject     -0.123169   0.098312   -1.253  0.210264
distribution     -0.871407   0.111186   -7.837  4.60e-15 ***
primaryoffyear    -0.249980   0.140975   -1.773  0.076191 .
repgov            -0.123169   0.098312   -1.253  0.210264
repleg            -0.271830   0.113750   -2.390  0.016862 *
ideology          3.270143   0.895838    3.650  0.000262 ***
legprof           -0.785476   0.378578   -2.075  0.038004 *
time              -0.785476   0.378578   -2.075  0.038004 *
minordiv          -1.961033   0.943860   -2.078  0.037739 *
statcons          -1.961033   0.943860   -2.078  0.037739 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for Negative Binomial(9.2151) family taken to be 1)

Null deviance: 709.82  on 276  degrees of freedom
Residual deviance: 327.11  on 259  degrees of freedom
Today: Event Count Models

(27 observations deleted due to missingness)
AIC: 1035.1
Number of Fisher Scoring iterations: 1

Call:
`vglm(formula = ballot ~ citizen + (bunch of vars), family = negbinomial, zero = NULL)`

Coefficients:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Std. Error</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept):1</td>
<td>-1.519263</td>
<td>0.4888323</td>
<td>-3.1079</td>
</tr>
<tr>
<td>(Intercept):2</td>
<td>2.220329</td>
<td>0.3628234</td>
<td>6.1196</td>
</tr>
<tr>
<td>citizen</td>
<td>0.296601</td>
<td>0.0955332</td>
<td>3.1047</td>
</tr>
<tr>
<td>econ</td>
<td>-0.156014</td>
<td>0.0585289</td>
<td>-2.6656</td>
</tr>
<tr>
<td>statinit</td>
<td>3.309472</td>
<td>0.4426172</td>
<td>7.4771</td>
</tr>
<tr>
<td>consinit</td>
<td>0.704636</td>
<td>0.1573553</td>
<td>4.4780</td>
</tr>
<tr>
<td>sigs</td>
<td>-0.168122</td>
<td>0.0274846</td>
<td>-6.1170</td>
</tr>
<tr>
<td>unlimit</td>
<td>0.465719</td>
<td>0.2007656</td>
<td>2.3197</td>
</tr>
<tr>
<td>circdays</td>
<td>0.084737</td>
<td>0.0387444</td>
<td>2.1871</td>
</tr>
<tr>
<td>singlesubject</td>
<td>0.421770</td>
<td>0.1265742</td>
<td>3.3322</td>
</tr>
<tr>
<td>distribution</td>
<td>-0.871386</td>
<td>0.1112114</td>
<td>-7.8354</td>
</tr>
<tr>
<td>primaryoffyear</td>
<td>-0.249915</td>
<td>0.1410042</td>
<td>-1.7724</td>
</tr>
<tr>
<td>repgov</td>
<td>-0.123160</td>
<td>0.0983380</td>
<td>-1.2524</td>
</tr>
<tr>
<td>repleg</td>
<td>-0.271870</td>
<td>0.1137786</td>
<td>-2.3895</td>
</tr>
<tr>
<td>ideology</td>
<td>3.270317</td>
<td>0.8959919</td>
<td>3.6499</td>
</tr>
<tr>
<td>legprof</td>
<td>-0.785689</td>
<td>0.3786702</td>
<td>-2.0749</td>
</tr>
<tr>
<td>time</td>
<td>0.010891</td>
<td>0.0083383</td>
<td>1.3062</td>
</tr>
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<td>minordiv</td>
<td>2.235391</td>
<td>0.7250076</td>
<td>3.0833</td>
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<td>deficit</td>
<td>-1.961162</td>
<td>0.9440554</td>
<td>-2.0774</td>
</tr>
</tbody>
</table>

Number of linear predictors: 2
Names of linear predictors: log(mu), log(k)
Dispersion Parameter for negbinomial family: 1
Log-likelihood: -498.5379 on 535 degrees of freedom