

# Count Models

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## Today: Event Count Models

# Topics

- ▶ Poisson Distribution
- ▶ Different “Flavors” of Event Count Models

# Motivation

- ▶ Nature of the Response Variable,  $y$
- ▶ Non-negative integer,  $y = 0, 1, 2, \dots, k$
- ▶ Examples:
  - Coups d'Etat
  - Presidential Vetoes
  - Cabinet Terminations
  - Tornado Deaths
- ▶ Events occur in an interval of time.
- ▶ Frequently the case that  $\Pr(\text{Event})$  is small, but  $n$  time periods is large.
- ▶ Usually assume the observation period is fixed (i.e. it doesn't vary).

# Issues

- ▶ The response variable is “absolutely continuous.”
- ▶ Normal distribution does a poor job of describing DGP.
- ▶ Normal models, like OLS, may return negative predicted values.
- ▶ ... while having similar kinds of problems we saw wrt categorical variables.

# The Poisson

- ▶ Useful to consider the Poisson distribution.
- ▶ Poisson distribution describes the probability of events occurring within  $t$ ,  $t + 1$
- ▶ Probability function is given by:

$$\Pr(Y_i = y) = \frac{\exp(-\lambda_i)\lambda_i^{y_i}}{y_i!}$$

- ▶  $\lambda$  gives us the “expected number of counts.”
- ▶ The PDF is asymmetric for small  $\lambda$  (Why?)

```
Simulation: 1000 draws from a Poisson, lambda=1,5,20,100

samplesize<-1000
lambda_1<-sample(rpois(samplesize, 1))
hist(lambda_1, br=c(0,1,2,3,4,5,6,7,8,9,10), col="blue1")

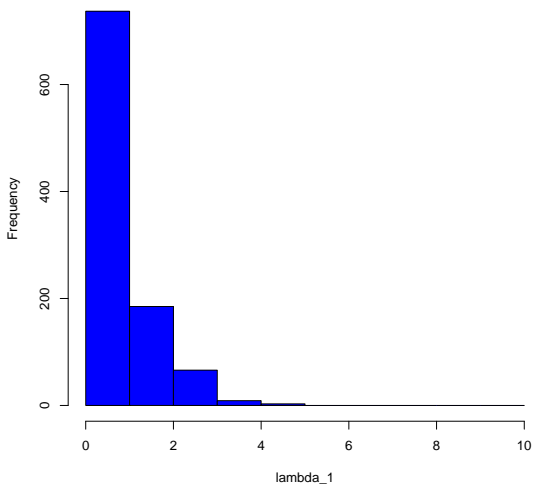
lambda_5<-sample(rpois(samplesize, 5))
hist(lambda_5, br=c(0:20), col="blue1")

lambda_20<-sample(rpois(samplesize, 20))
hist(lambda_20, br=c(0:50), col="blue1")

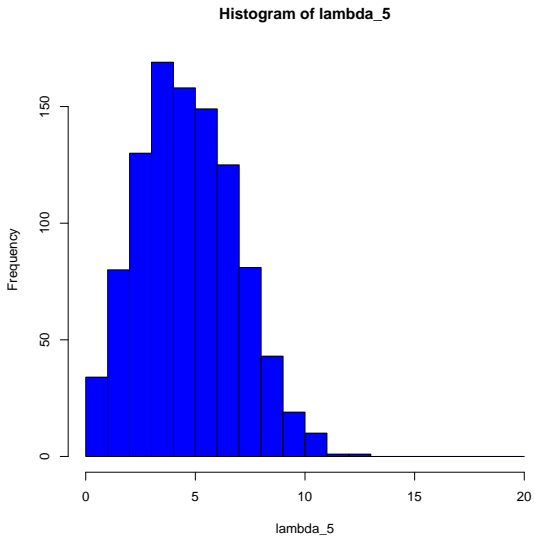
lambda_100<-sample(rpois(samplesize, 100))
hist(lambda_100, br=c(0:150), col="blue1")
```

$$\lambda = 1$$

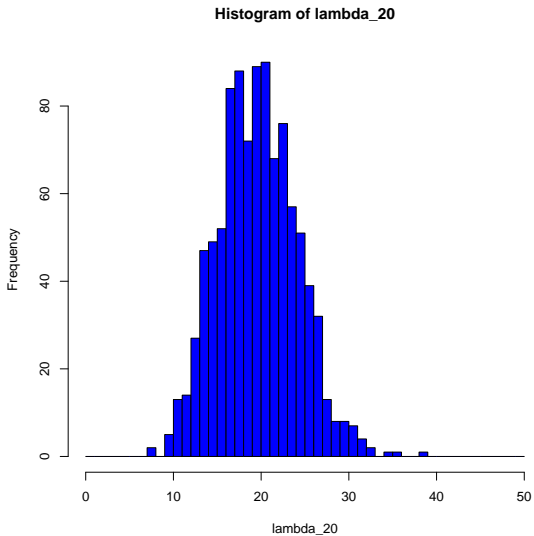
Histogram of lambda\_1



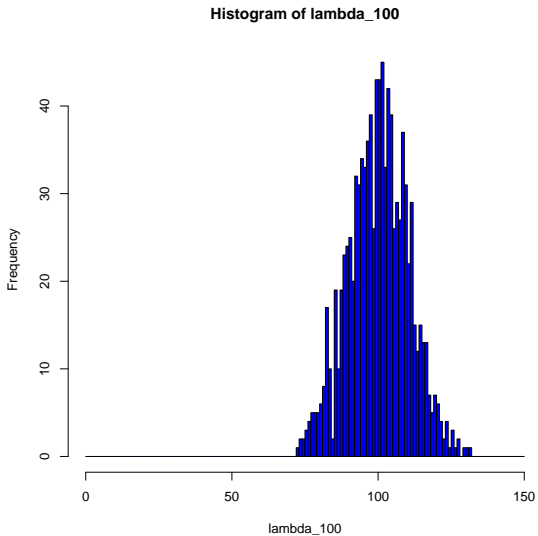
$$\lambda = 5$$



$$\lambda = 20$$



$$\lambda = 100$$



# Some Properties of Poisson

- ▶ This is a single parameter distribution.
- ▶ Implies:  $E(Y) = \lambda = \text{var}(\lambda)$
- ▶ Thus, the variance is increasing with the mean.
- ▶ We may be interested in modeling expected counts as a function of covariates.
- ▶ This is the Poisson Model.

# The Poisson Model

- ▶ Expectation as a function of covariates:

$$E[y_i | \mathbf{x}] = \lambda = \exp(\beta' \mathbf{x}_i)$$

- ▶ As probability:

$$\Pr(Y_i = y) = \frac{e^{-e^{\beta' \mathbf{x}_i}} (e^{\beta' \mathbf{x}_i})^{y_i}}{y_i!}$$

- ▶ As log-linear model (i.e. linear model for  $\log E[y_i | \mathbf{x}]$ ):

$$\log E[y_i | \mathbf{x}] = \beta' \mathbf{x}_i$$

- ▶ Cameron and Trivedi refer to the first two functions as the Poisson regression model.
- ▶ Likelihood:

$$\log L = \sum_{i=1}^n \{-\exp(\beta' \mathbf{x}_i) + y_i \beta' \mathbf{x}_i - \log y_i!\}$$

# Interpretation

- ▶ Interest often centers on “expected counts” given some covariate:

$$E[y_i | \mathbf{x}] = \lambda = \exp(\beta' \mathbf{x}_i)$$

- ▶ Or marginal effects:

$$\frac{\partial E[y_i | \mathbf{x}_i]}{\partial \mathbf{x}_i} = \exp(\beta' \mathbf{x}_i) \beta$$

# Illustration

- ▶ Use Fred Boehmke's data from 2005 *PRQ*.
- ▶ Response variable: count of ballot initiatives.
- ▶ Stata first; R second.

```
. poisson ballot citizen econ statinit consinit
  sigs unlimit circdays singlesu distribu primaryo repgov repleg i
> deology legprof time minordiv deficit, cluster(stateno)
```

```
Iteration 0:  log pseudolikelihood = -508.68756
Iteration 1:  log pseudolikelihood = -504.7026
Iteration 2:  log pseudolikelihood = -504.6913
Iteration 3:  log pseudolikelihood = -504.6913
Poisson regression      Number of obs   =       277
                        Wald chi2(17)      =       889.33
Log pseudolikelihood = -504.6913      Prob > chi2     =       0.0000
                                (Std. Err. adjusted for 23 clusters in stateno)
```

|  | ballot       | Coef.     | Robust Std. Err. | z     | P> z  | [95% Conf. Interval] |           |
|--|--------------|-----------|------------------|-------|-------|----------------------|-----------|
|  | citizen      | .2706867  | .0658299         | 4.11  | 0.000 | .1416624             | .3997711  |
|  | econ         | -.1365263 | .0462521         | -2.95 | 0.003 | -.2271788            | -.0458738 |
|  | statinit     | 3.234443  | .6568364         | 4.92  | 0.000 | 1.947068             | 4.521819  |
|  | consinit     | .6646655  | .1363439         | 4.87  | 0.000 | .3974363             | .9318947  |
|  | sigs         | -.1652336 | .0299768         | -5.51 | 0.000 | -.2239871            | -.1064802 |
|  | unlimit      | .4954682  | .180718          | 2.74  | 0.006 | .1412674             | .8496689  |
|  | circdays     | .0785881  | .0360028         | 2.18  | 0.029 | .008024              | .1491523  |
|  | singlesubj`t | .4259296  | .1301254         | 3.27  | 0.001 | .1708884             | .6809707  |
|  | distribution | -.8871728 | .1188417         | -7.47 | 0.000 | -1.120098            | -.6542474 |
|  | primaryoff`r | -.2958509 | .1292109         | -2.29 | 0.022 | -.5490997            | -.0426021 |
|  | repgov       | -.1339101 | .1015792         | -1.32 | 0.187 | -.3330018            | .0651816  |
|  | repleg       | -.2437057 | .1255485         | -1.94 | 0.052 | -.4897762            | .0023649  |
|  | ideology     | 3.140647  | .6233684         | 5.04  | 0.000 | 1.918867             | 4.362426  |
|  | legprof      | -.6318353 | .4315538         | -1.46 | 0.143 | -1.477665            | .2139946  |
|  | time         | .0122646  | .0103938         | 1.18  | 0.238 | -.008107             | .0326362  |
|  | minordiv     | 2.122436  | 1.019438         | 2.08  | 0.037 | .1243744             | 4.120497  |
|  | deficit      | -1.923051 | 1.178353         | -1.63 | 0.103 | -4.232581            | .3864794  |
|  | _cons        | -1.48057  | .6620717         | -2.24 | 0.025 | -2.778207            | -.1829339 |

Call:

```
glm(formula = ballot ~ citizen + econ + statinit + consinit +
     sigs + unlimit + circdays + singlesubject + distribution +
     primaryoffyear + reggov + repleg + ideology + legprof + time +
     minordiv + deficit, family = poisson())
```

Deviance Residuals:

|  | Min     | 1Q      | Median  | 3Q     | Max    |
|--|---------|---------|---------|--------|--------|
|  | -2.5581 | -1.1366 | -0.2113 | 0.6015 | 3.5198 |

Coefficients:

|                | Estimate | Std. Error | z value | Pr(> z ) |     |
|----------------|----------|------------|---------|----------|-----|
| (Intercept)    | -1.48057 | 0.44301    | -3.342  | 0.000832 | *** |
| citizen        | 0.27069  | 0.08063    | 3.357   | 0.000788 | *** |
| econ           | -0.13653 | 0.04891    | -2.792  | 0.005245 | **  |
| statinit       | 3.23444  | 0.40877    | 7.913   | 2.52e-15 | *** |
| consinit       | 0.66466  | 0.13958    | 4.762   | 1.92e-06 | *** |
| sigs           | -0.16523 | 0.02431    | -6.796  | 1.08e-11 | *** |
| unlimit        | 0.49547  | 0.17567    | 2.820   | 0.004797 | **  |
| circdays       | 0.07859  | 0.03494    | 2.250   | 0.024479 | *   |
| singlesubject  | 0.42593  | 0.11325    | 3.761   | 0.000169 | *** |
| distribution   | -0.88717 | 0.09667    | -9.177  | < 2e-16  | *** |
| primaryoffyear | -0.29585 | 0.12358    | -2.394  | 0.016663 | *   |
| reggov         | -0.13391 | 0.08047    | -1.664  | 0.096099 | .   |
| repleg         | -0.24371 | 0.09612    | -2.535  | 0.011233 | *   |
| ideology       | 3.14065  | 0.79799    | 3.936   | 8.29e-05 | *** |
| legprof        | -0.63184 | 0.32019    | -1.973  | 0.048462 | *   |
| time           | 0.01226  | 0.00711    | 1.725   | 0.084516 | .   |
| minordiv       | 2.12244  | 0.64540    | 3.289   | 0.001007 | **  |
| deficit        | -1.92305 | 0.81840    | -2.350  | 0.018785 | *   |

---  
 Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
 (Dispersion parameter for poisson family taken to be 1)

Null deviance: 928.24 on 276 degrees of freedom

Residual deviance: 405.42 on 259 degrees of freedom

(27 observations deleted due to missingness)

AIC: 1045.4

Number of Fisher Scoring iterations: 6

Call:

```
vglm(formula = ballot ~ citizen + econ + statinit + consinit +
      sigs + unlimit + circdays + singlesubject + distribution +
      primaryoffyear + repgov + repleg + ideology + legprof + time +
      minordiv + deficit, family = poissonff)
```

Pearson Residuals:

|         | Min     | 1Q       | Median   | 3Q      | Max    |
|---------|---------|----------|----------|---------|--------|
| log(mu) | -2.0047 | -0.90063 | -0.20443 | 0.68061 | 6.2293 |

Coefficients:

|                | Value     | Std. Error | t value |
|----------------|-----------|------------|---------|
| (Intercept)    | -1.480570 | 0.4429487  | -3.3425 |
| citizen        | 0.270687  | 0.0806334  | 3.3570  |
| econ           | -0.136526 | 0.0489062  | -2.7916 |
| statinit       | 3.234443  | 0.4087049  | 7.9139  |
| consinit       | 0.664665  | 0.1395794  | 4.7619  |
| sigs           | -0.165234 | 0.0243137  | -6.7959 |
| unlimit        | 0.495468  | 0.1756689  | 2.8205  |
| circdays       | 0.078588  | 0.0349338  | 2.2496  |
| singlesubject  | 0.425930  | 0.1132493  | 3.7610  |
| distribution   | -0.887173 | 0.0966693  | -9.1774 |
| primaryoffyear | -0.295851 | 0.1235732  | -2.3941 |
| repgov         | -0.133910 | 0.0804709  | -1.6641 |
| repleg         | -0.243706 | 0.0961210  | -2.5354 |
| ideology       | 3.140647  | 0.7979709  | 3.9358  |
| legprof        | -0.631835 | 0.3201900  | -1.9733 |
| time           | 0.012265  | 0.0071095  | 1.7251  |
| minordiv       | 2.122436  | 0.6453928  | 3.2886  |
| deficit        | -1.923051 | 0.8183928  | -2.3498 |

Number of linear predictors: 1

Name of linear predictor: log(mu)

```
(Default) Dispersion Parameter for poissonff family: 1  
Residual Deviance: 405.4232 on 259 degrees of freedom  
Log-likelihood: -504.6913 on 259 degrees of freedom  
Number of Iterations: 5
```

# Illustration

- ▶ Stata, use `poisson`
- ▶ R, I used `glm` and `vglm`
- ▶ Happily, all yield identical results.
- ▶ In R, I couldn't figure out how to suppress the intercept (which F.B. does in his article).  
It makes no difference though (just remove `statecons` from model).
- ▶ Interpretation?  
Would want to compute odds-like ratios:  $\exp(\beta x)$   
Expected counts for covariate profiles  
Marginal effects.

# Issues

- ▶ Recall that in Poisson, mean=variance.
- ▶ Poisson model presumes Poisson *process*:  
The probability of an event occurring within some period (usually measured temporally) is constant and independent of all previous events.
- ▶ If events tend to be “lumpy” (clustering) or non-independent, *dispersion* may exist.
- ▶ Convenient to think of “over” vs. “under” dispersion.

# Dispersion

- ▶ Overdispersion is common.  
Events are more prone for some sample elements compared to others.  
Event amass at different periods (probability is not constant).  
The occurrence of past events effect probability of future events.
- ▶ Underdispersion is apparently uncommon.  
Occurrence of past events decreases chances for future events.  
Evidently not widely found in social science literature.

# Dispersion

- ▶ Conceptually:

$$\sigma^2 = 1, \quad E(y_i) = \text{var}(y_i)$$

$$\sigma^2 > 1, \quad E(y_i) < \text{var}(y_i)$$

$$\sigma^2 < 1, \quad E(y_i) > \text{var}(y_i)$$

- ▶ The first is dispersion under the Poisson; the second is overdispersion; the third is underdispersion.
- ▶ Dispersion relates to efficiency of the estimator.
- ▶ Specifically, estimated parameters are consistent, but s.e. are inefficient and inconsistent.
- ▶ Leads to consideration of alternative modeling strategies.

# Over Dispersion

- ▶ Focus will be on over dispersion.
- ▶ Are there tests for overdispersion?
- ▶ Cameron and Trivedi (1998) propose a regression-based test:

$$\frac{(y_i - \hat{\lambda}_i)^2 - y_i}{\hat{\lambda}_i} = \alpha \hat{\lambda}_i + \epsilon_i$$
$$\frac{(y_i - \hat{\lambda}_i)^2 - y_i}{\hat{\lambda}_i} = \delta + \epsilon_i$$

where  $\hat{\lambda}_i = \exp(\beta' \mathbf{x}_i)$ . If Poisson dispersion holds, the mean function should have no effect on the variance function.

- ▶ Model-based test: negative binomial vs. Poisson.

# Negative Binomial Model

- ▶ What follows is adapted from Long (1997).
- ▶ The motivation:

$$\tilde{\lambda}_i = \exp(\beta' \mathbf{x}_i + \epsilon_i)$$

- ▶ The problem:

$$\begin{aligned}\tilde{\lambda}_i &= \exp(\beta' \mathbf{x}_i) \exp(\epsilon_i) \\ &= \lambda_i \exp(\epsilon_i) \\ &= \lambda_i \delta_i\end{aligned}$$

where  $\delta_i = \exp(\epsilon_i)$ .

# Negative Binomial Model

- ▶ We need structure on  $\delta_i$  in order to proceed.
- ▶ Convenient to assume  $E(\delta_i) = 1$ .
- ▶ For our purposes, this yields a nice result:

$$\begin{aligned}E(\tilde{\lambda}_i) &= E(\lambda_i \delta_i) \\ &= \lambda_i E(\delta_i) \\ &= \lambda_i.\end{aligned}$$

- ▶ And in probabilities:

$$\begin{aligned}\Pr(Y_i = y \mid \mathbf{x}, \delta_i) &= \frac{\exp(-\tilde{\lambda}_i) \tilde{\lambda}_i^{y_i}}{y_i!} \\ &= \frac{\exp(-\lambda_i \delta_i) (\lambda_i \delta_i)^{y_i}}{y_i!}\end{aligned}\tag{1}$$

- ▶ We now have a Poisson process.

# Negative Binomial Model

- ▶ The problem is,  $\delta_i$  is unknown and so we cannot directly estimate the previous function.
- ▶ What do we do?

$$\Pr(Y_i | \mathbf{x}_i) = \int_0^{\infty} [\Pr(y_i | \mathbf{x}_i \delta_i) \times g(\delta_i)] d\delta_i \quad (2)$$

- ▶ What we're doing here is integrating over values of  $\delta_i$ . In a sense, we're averaging over the probability of each  $\delta_i$
- ▶ From Long (1997, p. 231), suppose  $\delta_i$  has two values,  $d_1$  and  $d_2$ :

$$\begin{aligned} \Pr(Y_i | \mathbf{x}_i) &= [\Pr(y_i | \mathbf{x}_i \delta_i = d_1) \times \Pr(\delta_i = d_1)] \\ &+ [\Pr(y_i | \mathbf{x}_i \delta_i = d_2) \times \Pr(\delta_i = d_2)] \end{aligned}$$

# Negative Binomial Model

- ▶ In order to do all this, we need to specify a PDF for  $\delta_i$ .
- ▶ A convenient and commonly used distribution here is the gamma.

$$g(\delta_i) = \frac{\nu_i^{\nu_i}}{\Gamma(\nu_i)} \delta_i^{\nu_i-1} \exp(-\delta_i \nu_i) \quad (3)$$

- ▶  $\Gamma$  is the gamma function.
- ▶ Under the gamma,  $E(\delta_i) = 1$  (which is what we wanted) and  $\text{var}(\delta_i) = 1/\nu_i$ .

# Negative Binomial Model

- ▶ Solve equation 2 using equation 1 for  $\Pr(Y_i = y \mid \mathbf{x}, \delta_i)$  and equation 3 for  $g(\delta_i)$  and you obtain the negative binomial model:

$$\Pr(Y_i \mid \mathbf{x}_i) = \frac{\Gamma(y_i + \nu_i)}{y_i! \Gamma \nu_i} \left( \frac{\nu_i}{\nu_i + \lambda_i} \right)^{\nu_i} \left( \frac{\lambda_i}{\nu_i + \lambda_i} \right)^{y_i} \quad (4)$$

- ▶ Under this,  $E(y_i \mid \mathbf{x}_i) = \exp(\beta' \mathbf{x}_i)$  (which is good).
- ▶ The mean function is same as Poisson, but the variance is different.

$$\text{var}(y_i \mid \mathbf{x}_i) = \lambda \left( 1 + \frac{\lambda_i}{\nu_i} \right)$$

# Negative Binomial Model

- ▶ One thing left to do: put an identifying restriction on  $\nu_i$ .

$$\nu_i = \alpha^{-1}$$

is standard.

- ▶ By removing the subscript  $i$ , we've turned the variance into a constant.
- ▶ This gives a variance function of:

$$\text{var}(y_i | \mathbf{x}_i) = \lambda_i + \alpha \lambda_i^2$$

(Proof given in Cameron and Trivedi, Chapter 3)

- ▶  $\alpha$  is a dispersion parameter. As it approaches 0, the Poisson model is obtained.

# Negative Binomial Model

- ▶ Likelihood

$$L = \prod_{i=1}^N \frac{\Gamma(y_i + \alpha^{-1})}{y_i! \Gamma \alpha^{-1}} \left( \frac{\alpha^{-1}}{\alpha^{-1} + \lambda_i} \right)^{\alpha^{-1}} \left( \frac{\lambda_i}{\alpha^{-1} + \lambda_i} \right)^{y_i}$$

- ▶ Note: often  $\log(\alpha)$  is used in estimation; report  $\alpha$ , however.
- ▶ Interpretation: covariates have same interpretation as Poisson because mean function is the same. Standard interpretative tricks apply.
- ▶ Illustration using Fred Boehmke's data.

```

. nbreg ballot citizen econ statinit consinit
  sigs unlimit circdays singlesu distribu primaryo repgov repleg ideology
  legprof time minordiv deficit, cluster(staten0) nolog
Negative binomial regression          Number of obs   =       277
Dispersion = mean                    Wald chi2(17)   =       635.65
Log pseudolikelihood = -498.53791    Prob > chi2    =       0.0000
                                     (Std. Err. adjusted for 23 clusters in stateno)

```

| ballot       | Coef.     | Robust Std. Err. | z     | P> z  | [95% Conf. Interval] |           |
|--------------|-----------|------------------|-------|-------|----------------------|-----------|
| citizen      | .2965608  | .071057          | 4.17  | 0.000 | .1572917             | .4358299  |
| econ         | -.1559814 | .048059          | -3.25 | 0.001 | -.2501753            | -.0617875 |
| statinit     | 3.309358  | .6821106         | 4.85  | 0.000 | 1.972446             | 4.64627   |
| consinit     | .7045804  | .1454361         | 4.84  | 0.000 | .4195309             | .9896299  |
| sigs         | -.1681165 | .0294342         | -5.71 | 0.000 | -.2258065            | -.1104265 |
| unlimit      | .4657434  | .1805621         | 2.58  | 0.010 | .1118481             | .8196386  |
| circdays     | .0847279  | .035506          | 2.39  | 0.017 | .0151374             | .1543184  |
| singlesubj`t | .421773   | .1303808         | 3.23  | 0.001 | .1662313             | .6773146  |
| distribution | -.8714067 | .116634          | -7.47 | 0.000 | -1.100005            | -.6428081 |
| primaryoff`r | -.2499797 | .1272447         | -1.96 | 0.049 | -.4993746            | -.0005847 |
| repgov       | -.1231686 | .1046831         | -1.18 | 0.239 | -.3283437            | .0820064  |
| repleg       | -.2718297 | .1269838         | -2.14 | 0.032 | -.5207134            | -.0229461 |
| ideology     | 3.270144  | .633435          | 5.16  | 0.000 | 2.028634             | 4.511654  |
| legprof      | -.7854763 | .4351239         | -1.81 | 0.071 | -1.638303            | .0673509  |
| time         | .0108917  | .0097875         | 1.11  | 0.266 | -.0082914            | .0300749  |
| minordiv     | 2.235202  | .9640336         | 2.32  | 0.020 | .3457311             | 4.124673  |
| deficit      | -1.961033 | 1.146748         | -1.71 | 0.087 | -4.208619            | .2865523  |
| _cons        | -1.519194 | .6931931         | -2.19 | 0.028 | -2.877827            | -.1605602 |
| /lnalpha     | -2.220836 | .439864          |       |       | -3.082954            | -1.358719 |
| alpha        | .1085183  | .0477333         |       |       | .0458237             | .2569899  |

```

Call:
glm.nb(formula = ballot ~ citizen + (bunch of vars), init.theta = 9.21509710247016,
link = log)
Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.4223 -1.0039 -0.2209  0.5445  2.8634
Coefficients: (1 not defined because of singularities)
              Estimate Std. Error z value Pr(>|z|)
(Intercept)  -1.519194   0.488739  -3.108 0.001881 **
citizen       0.296561   0.095508   3.105 0.001902 **
econ        -0.155981   0.058512  -2.666 0.007680 **
statinit     3.309358   0.442546   7.478 7.55e-14 ***
consinit     0.704580   0.157327   4.478 7.52e-06 ***
sigs        -0.168117   0.027479  -6.118 9.48e-10 ***
unlimit      0.465744   0.200723   2.320 0.020323 *
circdays     0.084728   0.038738   2.187 0.028728 *
singlsubject 0.421773   0.126554   3.333 0.000860 ***
distribution -0.871407   0.111186  -7.837 4.60e-15 ***
primaryoffyear -0.249980   0.140975  -1.773 0.076191 .
repgov      -0.123169   0.098312  -1.253 0.210264
repleg      -0.271830   0.113750  -2.390 0.016862 *
ideology     3.270143   0.895838   3.650 0.000262 ***
legprof     -0.785476   0.378578  -2.075 0.038004 *
time         0.010892   0.008336   1.307 0.191378
minordiv     2.235202   0.724875   3.084 0.002045 **
deficit     -1.961033   0.943860  -2.078 0.037739 *
statcons    NA         NA         NA         NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for Negative Binomial(9.2151) family taken to be 1)
Null deviance: 709.82 on 276 degrees of freedom
Residual deviance: 327.11 on 259 degrees of freedom

```

(27 observations deleted due to missingness)

AIC: 1035.1

Number of Fisher Scoring iterations: 1

Call:

`vglm(formula = ballot ~ citizen + (bunch of vars), family = negbinomial, zero = NULL)`

Coefficients:

|                | Value     | Std. Error | t value |
|----------------|-----------|------------|---------|
| (Intercept):1  | -1.519263 | 0.4888323  | -3.1079 |
| (Intercept):2  | 2.220329  | 0.3628234  | 6.1196  |
| citizen        | 0.296601  | 0.0955332  | 3.1047  |
| econ           | -0.156014 | 0.0585289  | -2.6656 |
| statinit       | 3.309472  | 0.4426172  | 7.4771  |
| consinit       | 0.704636  | 0.1573553  | 4.4780  |
| sigs           | -0.168122 | 0.0274846  | -6.1170 |
| unlimit        | 0.465719  | 0.2007656  | 2.3197  |
| circdays       | 0.084737  | 0.0387444  | 2.1871  |
| singlesubject  | 0.421770  | 0.1265742  | 3.3322  |
| distribution   | -0.871386 | 0.1112114  | -7.8354 |
| primaryoffyear | -0.249915 | 0.1410042  | -1.7724 |
| reggov         | -0.123160 | 0.0983380  | -1.2524 |
| repleg         | -0.271870 | 0.1137786  | -2.3895 |
| ideology       | 3.270317  | 0.8959919  | 3.6499  |
| legprof        | -0.785689 | 0.3786702  | -2.0749 |
| time           | 0.010891  | 0.0083383  | 1.3062  |
| minordiv       | 2.235391  | 0.7250076  | 3.0833  |
| deficit        | -1.961162 | 0.9440554  | -2.0774 |

Number of linear predictors: 2

Names of linear predictors: log(mu), log(k)

Dispersion Parameter for negbinomial family: 1

Log-likelihood: -498.5379 on 535 degrees of freedom