POL 681: Answer Key for Problem Set 3

1. Substantive interpretation of each of the coefficients.

$\hat{b}_1$: Prior vote is positively related to the expected length of time that passes before a quality challenger enters a congressional race. For each 1 percent (scaled as .01) of the vote the incumbent received in the previous election, the expected length of time until a quality challenger emerges increases by .0856 weeks (note the answer is *not* 8.595 weeks because the covariate is scaled to lie between 0 and 1).

$\hat{b}_2$: An incumbent's campaign warchest is positively related to the expected duration of time that passes until a quality challenger emerges. For every $1000 increase in the value of the warchest, the expected length of time until a quality challenger emerges increase by about .015 weeks (that is 15.214*.001; note that a "one-unit" increase for this variable is equivalent to .000001 which would give an expected increase in the time-to-entry of a strong challenger equal to .000015214 weeks—for every dollar increase, the expected duration time increases by this amount).

$\hat{b}_3$: Southern congressional districts are associated with slightly lower expected times-to-entry for quality challengers. For a Southern district, the expected duration time decreases by about 2.1 weeks, compared to non-Southern districts.

$\hat{a}$: The intercept is not naturally interpretable. It says that if each of the covariates are 0 (which they cannot be jointly), then the expected duration time is about 44 weeks.

2. Mean square error is given by $SSError/(n - k - 1) = 548265.261/1372 = 399.61$.

3. The standard error of the estimate is given by the square root of the mean square error (also referred to as the root mean square error) and is $\sqrt{399.61} = 19.99$. This number tells us that the average deviation of the observed entry times from the predicted entry times is about plus or minus 19.99 weeks.

4. Using the formulae for the standard errors given in lecture notes, I find that the standard errors for each of the slope coefficients are given by

$$\begin{aligned} s.e.(\hat{b}_1) &= \frac{RMSE}{\sqrt{\sum(X_1 - \bar{X}_1)^2(1 - r^2_{X_1|X_2,X_3})}} \\ &= \frac{19.99}{\sqrt{(27.5)(.9746)}} = 3.861 \end{aligned}$$

for $\hat{b}_1$;

$$
\begin{aligned}
s.e.(\hat{b}_2) &= \frac{RMSE}{\sqrt{\sum(X_2 - \bar{X}_2)^2(1 - r^2_{X_2|X_1,X_3})}} \\
&= \frac{19.99}{\sqrt{(66)(.9768)}} = 2.49
\end{aligned}
$$

for $\hat{b}_2$; and

$$
\begin{aligned}
s.e.(\hat{b}_3) &= \frac{RMSE}{\sqrt{\sum(X_3 - \bar{X}_3)^2(1 - r^2_{X_3|X_1,X_2})}} \\
&= \frac{19.99}{\sqrt{(257.13)(.9977)}} = 1.25
\end{aligned}
$$

for $\hat{b}_3$. The sum of the squared deviations were obtained by multiplying the variances of each covariate by $n - 1$ and the auxiliary regressions were given in the problem.

5. The 95 percent confidence intervals for each coefficient are computed as follows. The critical t value for a confidence coefficient of .95 on 1372 degrees of freedom is 1.96. Using the standard errors computed above, we find that the 95 percent confidence interval for each slope coefficient is given by:

$$
\begin{aligned}
100(1 - \alpha)\% c.i. &= 8.595 \pm 7.57; \\
100(1 - \alpha)\% c.i. &= 15.214 \pm 4.88; \\
100(1 - \alpha)\% c.i. &= -2.098 \pm 2.45;
\end{aligned}
$$

for $\hat{b}_1, \hat{b}_2$, and $\hat{b}_3$ respectively.

6. The interpretation of these intervals is as follows: in repeated samples of size n from the same population, 95 percent of these samples will find the true population parameters (i.e. $\beta_1, \beta_2, \beta_3$) residing in the given interval. Another way to interpret them is that in the long run, 95 percent of all samples of the same size from the same population will yield intervals containing the true population parameter.

7. If an incumbent's warchest as no relationship to the time-to-entry of a quality challenger, then the hypothesis under this condition, called the null, is

$$H_o : \beta_2 = 0,$$

and the two-tailed alternative is

$$H_a : \beta_2 \neq 0.$$

8. Under the "confidence interval" approach to hypothesis testing, we reject the null hypothesis if the null condition is covered by the confidence interval. Let $\beta^* = 0$ denote the condition of the null. The 95 percent confidence interval around the warchest estimate ($\hat{b}_2$) has the lower bound of about 10 and an upper bound of about 20. Since β^* does not fall in this interval, we can reject the null and accept the alternative hypothesis with 95 percent confidence. Only 5 percent of all samples of the same size from the same population will give an interval containing β^*.

9. Treating the null condition as $\beta_1 = 0$, we define $\beta^* = 0$. The 95 percent confidence interval around $\hat{b}_3$ is given by:

$$\beta^* - 2.45 \leq \hat{b}_3 \leq \beta^* + 2.45.$$

We see that our coefficient estimate for $\hat{b}_2 = -2.098$. Since the coefficient falls *inside* the interval computed based on $\beta^* = 0$, we *cannot reject* the null hypothesis. Analogously, we see that the t statistic for this estimate is $-2.098/1.25) = -1.68$. The p-value for a $t = 1.68$ (taking the absolute value) is less than 1.96, which is the critical t value for 1372 degrees of freedom for $\alpha = .05$ (two-tail) test. Since the critical t for our test is less than the required critical t, we cannot reject the null hypothesis.

10. Let $\beta^* = 0$, as before. Now if $\alpha = .10$, then the 90 percent confidence interval around $\hat{b}_3$ is given by:

$$\beta^* - 2.056 \leq \hat{b}_3 \leq \beta^* + 2.056.$$

Since $\hat{b}_3 = -2.098$ falls *outside* of the interval based on $\beta^* = 0$, we *can* reject the null hypothesis with 90 percent confidence. Another way of seeing the same thing goes like this. The 90 percent confidence interval around $\hat{b}_3$ is -2.098 ± 2.056. If $\beta^* = 0$, then we can reject the null hypothesis with 90 percent confidence. This decision is based on the fact that β^* falls outside the the interval. The first test gives the "test-of-significance" approach; the latter test gives the "confidence interval" approach.

When compared to question 9, this test illustrates what happens when you decrease α. The confidence interval "shrinks," but so does the confidence coefficient. Another way to perform the exact same test is to compute the t value. We see that the t is -1.68. Consulting a t table, we find that the critical t for a two-tail test with $\alpha = .10$ on 1372 degrees of freedom is 1.65. Since our t value exceeds this critical value (in absolute value terms), we can reject the null with 90 percent confidence.

11. Under a 1-tail test, the alternative hypothesis for this problem is $H_a : \beta_3 < 0$. The critical t value for a 1-tail test with $\alpha = .05$ on 1372 degrees of freedom is 1.65. Since our t value of 1.68 (absolute value) exceeds this critical value, we can reject the null hypothesis with 95 percent confidence (1-tail test). The conclusion differs from the answer given in question 9 because the 1-tail test is

a less conservative test. As the t value for this test is "right on the edge" of statistical significance, it exceeds the critical t for a 1-tail test, but does not for a 2-tail test. This illustrates the point that you can come to different conclusions depending on the kind of test, 1 or 2-tail, you perform.

12. The F statistic is given by the MSR/MSE, where the $MSR = SSReg/(n-k-1)$. For this model, the F is 16.77. The p-value for an F distribution with 3 and 1372 degrees of freedom is less than .00001. This is a test that each of the slope coefficients are jointly 0. We can reject the null hypothesis.

Cabinet Data

1. Substantive interpretation of each of the coefficients.

$\hat{b}_1$: Support in a country for extremist parties is negatively related to the expected length of time a cabinet government stays in office. For a 1 percent increase in support of extremist parties, the expected duration of a cabinet government decreases by about .54 months.

$\hat{a}$: Since the polarization variable has a meaningful 0 point, the intercept is interpreted as giving us the expected duration time when there is 0 support for extremist parties in a country. This amounts to about 26.65 weeks.

2. The critical t-value for $\alpha = .05$ on 312 degrees of freedom is about 1.96. Given the standard errors, the 95 percent confidence interval for $\hat{b}_1$ is $-.537 \pm .12$ and for $\hat{a}$ is 26.652 ± 2.34. The interpretation of these intervals is as follows: in repeated samples of size n from the same population, 95 percent of these samples will find the true population parameters (i.e. α, β_1) residing in the given interval. Another way to interpret them is that in the long run, 95 percent of all samples of the same size from the same population will yield intervals containing the true population parameter.

3. The t-value for $\hat{b}_1$ under the condition of the null is -8.95. Since the critical t value for $\alpha = .05$ on 312 d.f. is 1.96 (two-tail), we can reject the null with 95 percent confidence. We come to this decision because our t-value is well above the critical value of 1.96. Note that if we had performed a 1-tail test, the critical t would have been about 1.65, and so we would have come to the same conclusion.