POL 681 Problem Set 3: Hypothesis Testing and Uncertainty

Objective: The objective of this problem set is to assess your ability to derive standard errors, confidence intervals, and goodness-of-fit indices for the traditional linear model.

Directions: Please answer each of the questions using the information provided. SHOW ALL WORK.

Warchest Data

For the following questions, the definition of the variables are as follows (note that these data are used in an article by Jan Box-Steffensmeier in the May 1996 *American Journal of Political Science*. Because Jan uses a different kind of statistical model in her analysis, the results presented below will not be identical to her results. Keep in mind that the purpose of using these data are *solely* for learning purposes. Later in the semester, we will learn about models more appropriate to these kinds of data. Here are the data:

Y denotes the length of time until a "high quality" challenger enters a House race against an incumbent. The scale of the dependent variable is in terms of the number of weeks that pass until a high-quality challenger emerges. The minimum is 1, denoting 1 week; the maximum is 90, denoting 90 weeks. I denote this variable as *Time-to-Entry*.

X_1 denotes the prior vote the incumbent received (scaled between 0 and 1; the minimum value is .5 (denoting 50 percent) and the maximum value is 1 (denoting 100 percent)). I denote this variable as *Prior Vote*.

X_2 denotes the incumbent's "warchest"; that is, the amount of money the incumbent has in reserve to use at his or her discretion (scaled in millions of dollars). The minimum value is .00069, which corresponds to \$690; the maximum value is 1.688, which corresponds to \$1,688,000. I denote this variable as *Warchest*.

X_3 is a dummy variable denoting whether or not the incumbent is in a Southern state (1 denotes South, 0 denotes non-South). I denote this variable as *South*.

The following descriptive statistics for the data will be helpful to have:

$$
\begin{aligned}
n &= 1376 & & \\
\bar{Y} &= 52.547 & s^2(Y) &= 413.359 \\
\bar{X}_1 &= .732 & s^2(X_1) &= .020 \\
\bar{X}_2 &= .206 & s^2(X_2) &= .048 \\
& & s^2(X_3) &= .187.
\end{aligned}
$$

Additionally, I estimated the auxiliary regression to obtain the r^2 from the following models:

$$X_1 = a + bX_2 + bX_3; r^2 = .0254$$
$$X_2 = a + bX_1 + bX_3; r^2 = .0232$$
$$X_3 = a + bX_1 + bX_2; r^2 = .0023$$

A regression model is estimated, giving us

$$\hat{Y} = 43.644 + 8.595(X_1) + 15.214(X_2) - 2.098(X_3).$$

From this model, the following variance components were obtained:

$$\sum (Y_i - \hat{Y}_i)^2 = 548265.261$$
$$\sum (\hat{Y}_i - \bar{Y})^2 = 20103.762.$$

Using the information given above, please answer the following questions:

1. What is the substantive interpretation of each of the regression coefficients? (16 points)

2. What is the mean square error for this model? (2 points)

3. What is the standard error of the estimate for this model? How is it interpreted? (4 points)

4. What are the standard errors of the regression coefficients (note that I have already computed the auxiliary regressions for you; this information is given to you above)? (12 points)

5. Compute the 95 percent confidence intervals for each coefficient. (12 points)

6. What is the interpretation of these intervals? (4 points)

7. Specify the null hypothesis for b_2 under the assumption that *Warchest* has no impact on *Time-to-Entry*. Specify the two-tail alternative to the null. (4 points)

8. Using the "confidence interval" approach to hypothesis testing, is the *Warchest* coefficient statistically significant? (Explain your answer and show any necessary work). (5 points)

9. Using the "test-of-significance" approach and using a two-tail test, evaluate the null hypothesis that the coefficient for *South* is equal to 0. For this test, let $\alpha = .05$. What does your test show? (Explain your answer and show any necessary work.) (5 points)

10. Re-do question 9, except now set $\alpha = .10$. What does your test show? (Explain your answer and show any necessary work.) (5 points)

11. Suppose that your alternative hypothesis for the *South* variable was that its coefficient was *less than 0*. Provide a one-tail hypothesis test setting $\alpha = .05$. What does your test show? Do the conclusions differ between this test and the test provided in question 9? (8 points)

12. Provide an F test for this model. What does the F-test tell you? (6 points)

Cabinet Data

The following questions use data on the length of time a cabinet government "survives" or persists, until it is terminated. These data are cross-national. These data have been widely used in the literature, but again, the purposes here are pedagogical. (To see articles where these data have been used for substantive analysis, consult: King, et. al. (1990 *AJPS*, Aug.); Warwick (1992 *AJPS,* Nov.).

For our purposes, we are interested in the following bivariate model:

$$Cabinet\ Duration = a + b_1(Polarization),$$

where *Cabinet Duration* denotes the number of months a cabinet government survives until it falls. This variable ranges from .5, denoting 1/2 a month, to 59, denoting 59 months. *Polarization* measures the support in the country for extremist political parties. This variable ranges from 0, denoting 0 percent, to 43, denoting 43 percent support. It is hypothesized that polarization will be negatively related to cabinet duration: the more support there is for extremist parties, the more difficult it will become for the governing party to bargain and hence, maintain a government. The sample size is $n = 314$.

The following information was produced from the regression:

$$\begin{aligned} \hat{a} &= 26.652 \qquad & s.e.(\hat{a}) &= 1.189 \\ \hat{b_1} &= -.537 \qquad & s.e.(\hat{b_1}) &= .060 \\ MSE &= 182.931. \end{aligned}$$

Further, note that the variance of *Polarization* is 164.2001, and so the sum of the squared mean deviations is $164.2001 * 313 = 51394.631$. Finally, the mean of *Polarization* is 15.290.

Using this information, answer the following questions:

1. What is the substantive interpretation of each coefficient (note that polarization has a meaningful 0 point)? (8 points)

2. Compute the 95 percent statement confidence interval for $\hat{a}$ and $\hat{b_1}$. How

are they interpreted? (8 points)

3. Using a t test, if the null is that the *Polarization* coefficient is 0, then is the coefficient statistically significant at the .05 level? (Explain your answer and show any work.) (5 points)