Order Matters (?): Alternatives to Conventional Practices for Ordinal Categorical Response Variables

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Ordinal Response Variables

- Ordinal scales are prevalent in survey research
- Frequently used as response variables
- Interest in $Y = f(\mathbf{x}'\beta)$
- Normal-theory methods commonly applied (OLS)
- Cumulative link models also applied
- Either strategy often works poorly

Common Issues That Arise

- Equal-interval scoring usually unrealistic
- "Parallel" regression assumption frequently will not hold
- Ordinality may not exist, conditional on covariates $\mathbf{x}$
- Alternative contrasts may be of interest
- Several alternative models are considered

Utility Motivation

- Assume Y is a discretized measure of Y^*
- Postulate $Y_i^* = \alpha + \mathbf{x}'\beta + \epsilon_i$
- Cut Point Rule:

$$
\begin{aligned}
Y = 1 &\equiv Y^* \leq \alpha_1 \\
Y = 2 &\equiv \alpha_1 < Y^* \leq \alpha_2 \\
Y = 3 &\equiv \alpha_2 < Y^* \leq \alpha_3 \\
Y = 4 &\equiv Y^* > \alpha_3
\end{aligned}
$$

- Specify C.D.F. for ϵ (logistic, standard normal, cloglog)
- With logistic, proportional odds is obtained.

Proportional Odds Model

- Gives rise to:

$$\Pr(Y \leq y_j \mid \mathbf{x}) = \frac{\exp(\alpha_j - \mathbf{x}'\beta)}{1 + \exp(\alpha_j - \mathbf{x}'\beta)}$$

- Linear Model for log-odds:

$$\log\left[\frac{\Pr(Y \leq y_j \mid \mathbf{x})}{\Pr(Y > y_j \mid \mathbf{x})}\right] = \alpha_j - \mathbf{x}'\beta, \quad j = 1, 2, \ldots j - 1$$

- Proportional Odds Property:

$$\frac{\exp(x_1\beta)}{\exp(x_2\beta)} = \exp\{(x_1 - x_2)'\beta\}$$

Standard Practice

- Assume Y is equal-interval scored and use normal-theory methods (OLS)
- Apply cumulative link model but rarely evaluate prop. odds assumption
- Or do both, conduct “eyeball test”, and report OLS

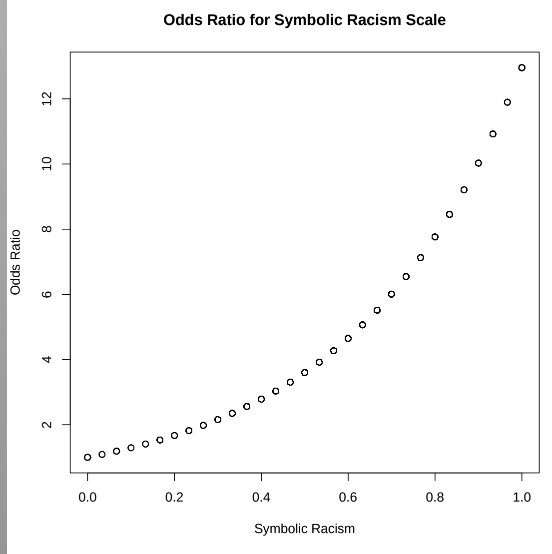
Application

Table 1: Support for Affirmative Action

OLS and Proportional Odds Estimates

Variable	**Estimate**	
	OLS	Proportional Odds
Symbolic Racism	1.375	2.562
	(0.111)	(0.207)
Racial Prejudice	0.383	0.728
	(0.193)	(0.356)
Education	0.259	0.510
	(0.102)	(0.182)
Ideology	0.066	0.117
	(0.027)	(0.047)
Female	0.025	0.075
	(0.049)	(0.088)
Intercept	1.479	
	(0.127)	
α_1		0.652
		(0.233)
α_2		1.888
		(0.236)
α_3		3.268
		(0.245)
n	1744	1744
log-likelihood:		-2289.518

Symbolic Racism



- Odds: $\exp(2.562) = 12.96$
- Odds Ratios Invariant to the Cut Points
- and they are proportional
- . . . but does the property hold?

Proportionality Tests

Table 2: Testing the Proportional Odds Assumption

Variable	**Coefficient**		
	$y > 1$	$y > 2$	$y > 3$
Symbolic Racism	1.822	2.458	3.037
Racial Prejudice	1.384	0.219	0.664
Education	0.209	0.496	0.651
Ideology	0.116	0.105	0.139
Female	-0.118	0.144	0.052
Constant	-0.279	-1.651	-3.645
Variable	χ^2	$p > \chi^2$	**df**
All	29.33	0.001	10
Symbolic Racism	12.37	0.002	2
Racial Prejudice	8.15	0.017	2
Education	2.29	0.319	2
Ideology	0.33	0.849	2
Female	5.79	0.055	2

Nonproportional Odds

- Generalized Model: $\log\left[\frac{\Pr(Y \leq y_j | \mathbf{x})}{\Pr(Y > y_j | \mathbf{x})}\right] = \alpha_j - \mathbf{x}'\beta_{\mathbf{j}}, \quad j = 1, 2, \ldots j - 1$

- Partial Proportional Odds:
$\log\left[\frac{\Pr(Y \leq y_j | \mathbf{x})}{\Pr(Y > y_j | \mathbf{x})}\right] = -\alpha_j - \mathbf{x}'\beta - \mathbf{t}'\gamma_{\mathbf{j}}, \quad j = 1, 2, \ldots j - 1$

- Restricted Generalized Logit:
$\log\left[\frac{\Pr(Y \leq y_j | \mathbf{x})}{\Pr(Y > y_j | \mathbf{x})}\right] = \alpha_j + \mathbf{x}'\beta + \mathbf{z}'\zeta_{\mathbf{j}}, \quad j = 1, 2, \ldots j - 1$

Table 3: Support for Affirmative Action

Partial Proportional Odds (UPP Model)

Variable	Coefficient	Standard Error
Symbolic Racism	1.788	(0.285)
γ_2	0.800	(0.257)
γ_3	1.345	(0.358)
Racial Prejudice	1.563	(0.495)
γ_2	-1.437	(0.440)
γ_3	-0.811	(0.586)
Female	-0.098	(0.125)
γ_2	0.277	(0.111)
γ_3	0.147	(0.145)
Education	0.479	(0.182)
Ideology	0.122	(0.047)
α_1	.273	(.22)
α_2	-1.635	(.203)
α_3	-3.263	(.236)
n	1744	
log-likelihood	-2274.293	

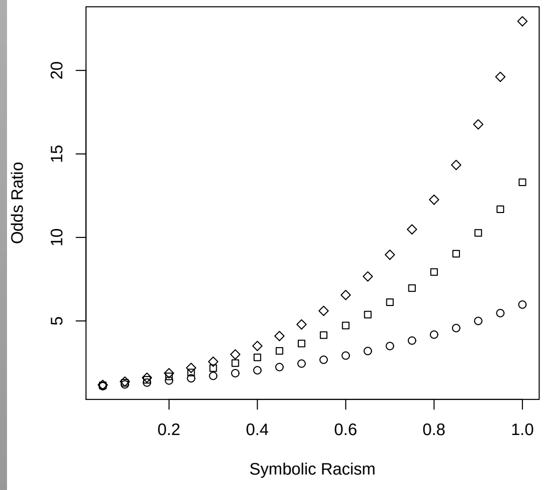
Application: Restricted Generalized Ordinal

Table 3a: Support for Affirmative Action

Partial Proportional Odds (Generalized Ordinal Logit)

Variable	**Coefficient**		
	C_1	C_2	C_3
Symbolic Racism	1.788 (0.285)	2.588 (0.245)	3.133 (0.280)
Racial Prejudice	1.563 (0.495)	0.127 (0.421)	0.752 (0.462)
Education	0.478 (0.182)	0.478 (0.182)	0.478 (0.182)
Ideology	0.122 (0.047)	0.122 (0.047)	0.122 (0.047)
Female	-0.098 (0.125)	0.180 (0.101)	0.049 (0.113)
Constant	-0.509 (0.283)	-1.698 (0.261)	-3.639 (0.299)
n	1744		
log-likelihood	-2274.293		

Symbolic Racism Redux



- Odds vary over cut points
- "Parallel Regression" does not hold
- Fit of partial prop. odds superior to proportional odds

Alternative Modeling Strategies

- Sometimes ordinality will not hold (or will not be of primary interest)
- Different contrasts may be of interest
- Leads to consideration of alternative models

Multinomial Models

- Baseline Category Logit: $\log\left[\frac{\Pr(Y=y_j|\mathbf{x})}{\Pr(Y=1|\mathbf{x})}\right] = \alpha_j + \mathbf{x}'\beta_{\mathbf{j}}, \quad j = 1, 2, \ldots j-1$

- Adjacent Category Logit: $\log\left[\frac{\Pr(Y=y_{j+1}|\mathbf{x})}{\Pr(Y=y_j|\mathbf{x})}\right] = \alpha_j + \mathbf{x}'\beta_{\mathbf{j}}, \quad j = 1, 2, \ldots j-1$

- Anderson's Stereotype Logit: $\log\left[\frac{\Pr(Y=y_j|\mathbf{x})}{\Pr(Y=1|\mathbf{x})}\right] = \alpha_j + \mathbf{x}'\phi_j\beta, \quad j = 1, 2, \ldots j-1$

- Latter gives a test of ordinality: $1 = \phi_1 > \phi_2 > \ldots \phi_J = 0$

Application: ACL

Table 4:

Adjacent Category Logit

Variable	**Coefficient**		
	2 vs 1	3 vs 2	4 vs 3
Moral Traditionalism	-0.812 (0.776)	-1.517 (0.639)	-1.752 (0.742)
Group Difference	-0.010 (0.096)	-0.028 (0.076)	-0.001 (0.088)
Hispanic Traits	-0.417 (0.948)	-1.092 (0.779)	-1.928 (0.876)
Ideology	0.490 (0.317)	0.031 (0.262)	0.027 (0.304)
Economic Evaluation	-0.324 (0.402)	-0.179 (0.336)	-0.714 (0.410)
Constant	1.161 (0.748)	2.235 (0.603)	1.353 (0.639)
n	774		
log-likelihood	−981.839		

Contrasts are for Adjacent Categories

- Consider the Moral Tradtionalism Scale
- C2 vs. C1: .44
- C3 vs. C2: .22
- C4 vs. C3: .17
- Odds are decreasing over each adjacent category
- Note that ACL=BCL: No statistical adjudication is possible
- Just the interpretation differs: ordinality is preserved through local logits
- ...though ordinality *need* not hold

Application: Stereotype

Table 5:

Stereotype Logit Model

Variable	**Coefficient**
Moral Traditionalism	4.111 (0.852)
Group Difference	0.044 (0.093)
Hispanic Traits	3.444 (0.990)
Ideology	-0.410 (0.328)
Economic Evaluation	1.092 (0.437)
α_1	-4.725 (0.825)
α_2	-3.660 (0.726)
α_3	-1.382 (0.624)
ϕ_1	1
ϕ_2	0.848 (0.131)
ϕ_3	0.507 (0.102)
ϕ_4	0
n	774
log-likelihood	-983.586

Interpretation?

- Ordinality condition holds (restrictions could be applied in some instances)
- Odds Ratios are in reference to baseline category ($Y = 4$)
- C1 vs. C4: 61 (Constrast of Scale Extremes)
- C2 vs. C4: 33
- C3 vs. C4: 8
- Model is not equivalent to BCL (unless $K = 1$)
- It is reduced rank BCL
- LR Test suggests BCL does not fit better

Conclusions

- Applied work frequently makes assumptions that will not hold
- A lot of potentially interesting information is ignored
- Different models yield different contrasts
- These alternatives may be more consistent with some theories of political behavior