$$h(t \mid \mathbf{x}) = \lim_{\Delta t \to 0} \frac{\Pr(t \leq T \leq t + \Delta t \mid T \geq t, \mathbf{x})}{\Delta t} \tag{1}$$

Exponential

$$h(t) = \lambda \qquad t > 0, \lambda > 0, \tag{2}$$

$$S(t) = \exp^{-\lambda(t)}, \tag{3}$$

$$f(t) = \lambda(t) \exp^{-\lambda(t)}. \tag{4}$$

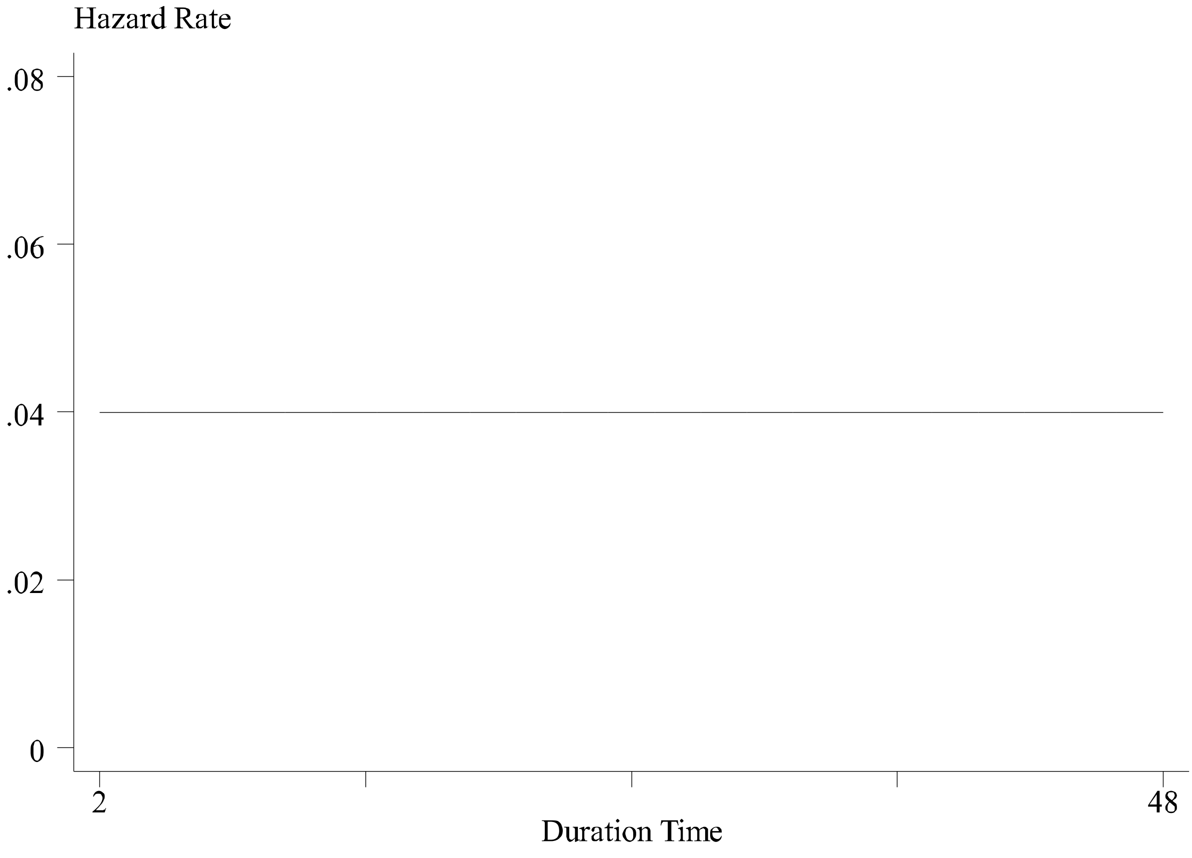

Figure 1: *This figure graphs a typical example of the exponential hazard rate.*

Weibull

$$h(t) = \lambda p(\lambda t)^{p-1} \qquad t > 0, \lambda > 0, p > 0, \tag{5}$$

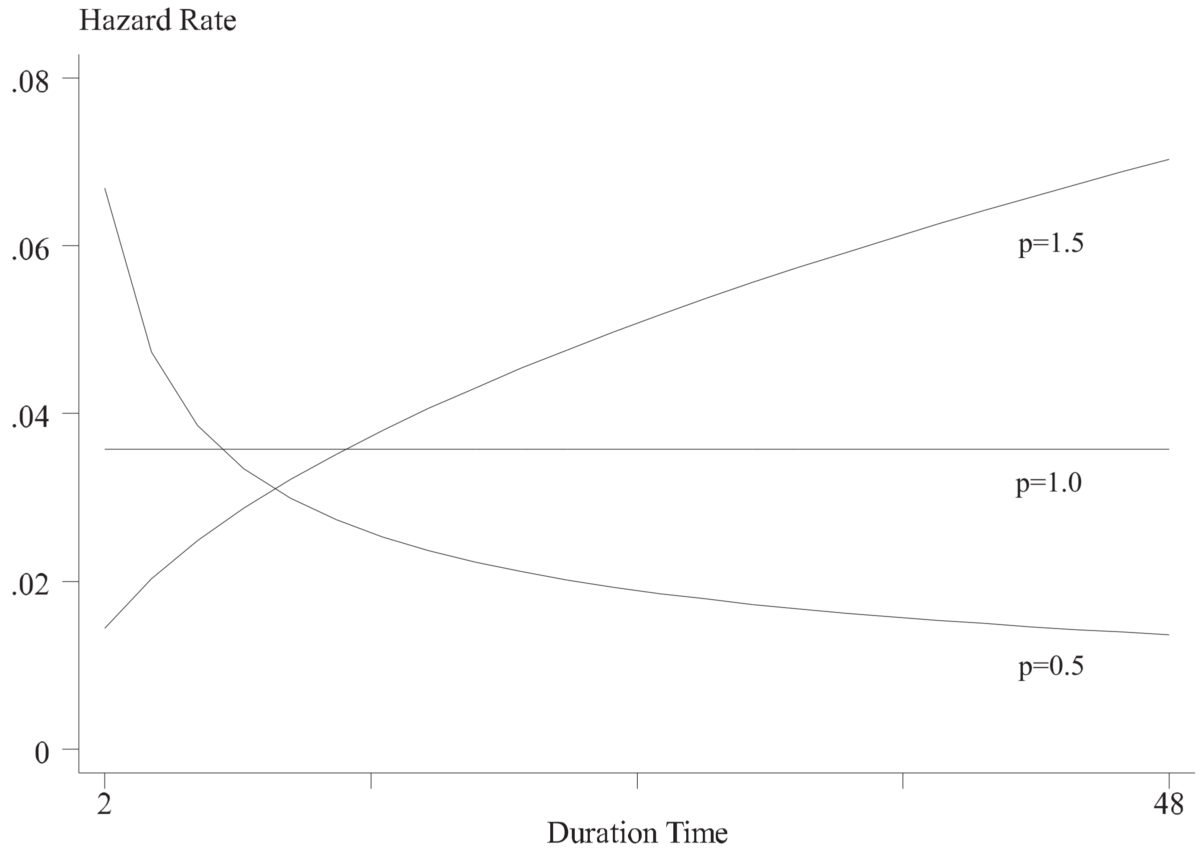

Figure 2: *This figure graphs three typically shaped Weibull hazard rates. Note the monotonicity of the Weibull hazard; note also that when the shape parameter is 1, the exponential hazard is obtained.*

$$S(t) = \exp^{-(\lambda t)^p}, \tag{6}$$

$$f(t) = \lambda p(\lambda t)^{p-1} \exp^{-(\lambda t)^p}. \tag{7}$$

A Sidenote: Accelerated Failure Time Parameterization

$$\log(T) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_j x_{ij} + \sigma\epsilon, \tag{8}$$

and in vector notation,

$$\log(T) = \beta_j' \mathbf{x} + \sigma\epsilon \tag{9}$$

$$h(t \mid \mathbf{x}) = h_{0t} \exp(\alpha_1 x_{i1} + \alpha_2 x_{i2} + \ldots + \alpha_j x_{ij}), \tag{10}$$

P.H. Parm.	A.F.T. Parm.	Relationship Between Parameters	Interp. of P.H. Parm.	Interp. of A.F.T. Parm.
α	β	$\beta = \frac{-\alpha}{p}$	$+\alpha \equiv\ \uparrow h(t \mid x_{ij})$	$+\beta \equiv\ \uparrow \log(T)$
		$\alpha = -\beta p$	$-\alpha \equiv\ \downarrow h(t \mid x_{ij})$	$-\beta \equiv\ \downarrow \log(T)$
p	σ	$\sigma = \frac{1}{p}$	$p > 1 \equiv\ \uparrow h(t \mid x_{ij})$	$\sigma > 1 \equiv\ \downarrow h(t \mid x_{ij})$
		$p = \frac{1}{\sigma}$	$p < 1 \equiv\ \downarrow h(t \mid x_{ij})$	$\sigma < 1 \equiv\ \uparrow h(t \mid x_{ij})$

The Log-Logistic and Log-Normal Models

$$\log(T) = \beta_j' \mathbf{x} + \sigma\epsilon. \tag{11}$$

Log-Logistic:

$$h(t) = \frac{\lambda p(\lambda t)^{p-1}}{1 + (\lambda t)^p} \tag{12}$$

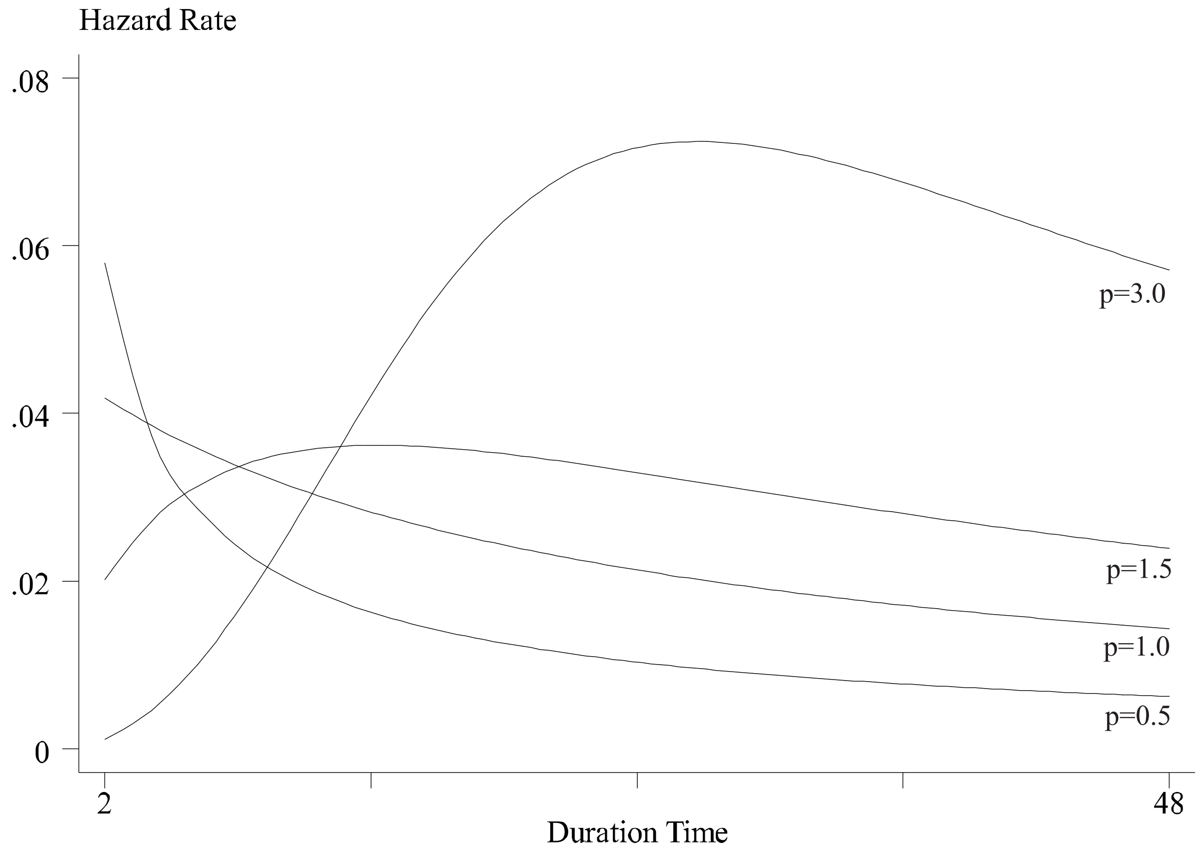

Figure 3: *This figure graphs some typically shaped hazard rates for the log-logistic model.*

$$S(t) = \frac{1}{1 + (\lambda t)^p}, \tag{13}$$

$$f(t) = \frac{\lambda p(\lambda t)^{p-1}}{(1 + (\lambda t)^p)^2}, \tag{14}$$

Log-Normal:

$$S(t) = 1 - \mathbf{\Phi}\Big(\frac{\log(t) - \beta'\mathbf{x}}{\sigma}\Big), \tag{15}$$

$$f(t) = \frac{1}{\sigma\sqrt{(2\pi)}} t^{-1} \exp\Big[- \frac{1}{2}\Big(\frac{\log(t) - \beta'\mathbf{x}}{\sigma}\Big)^2\Big], \tag{16}$$

$$h(t) = \frac{f(t)}{S(t)}. \tag{17}$$

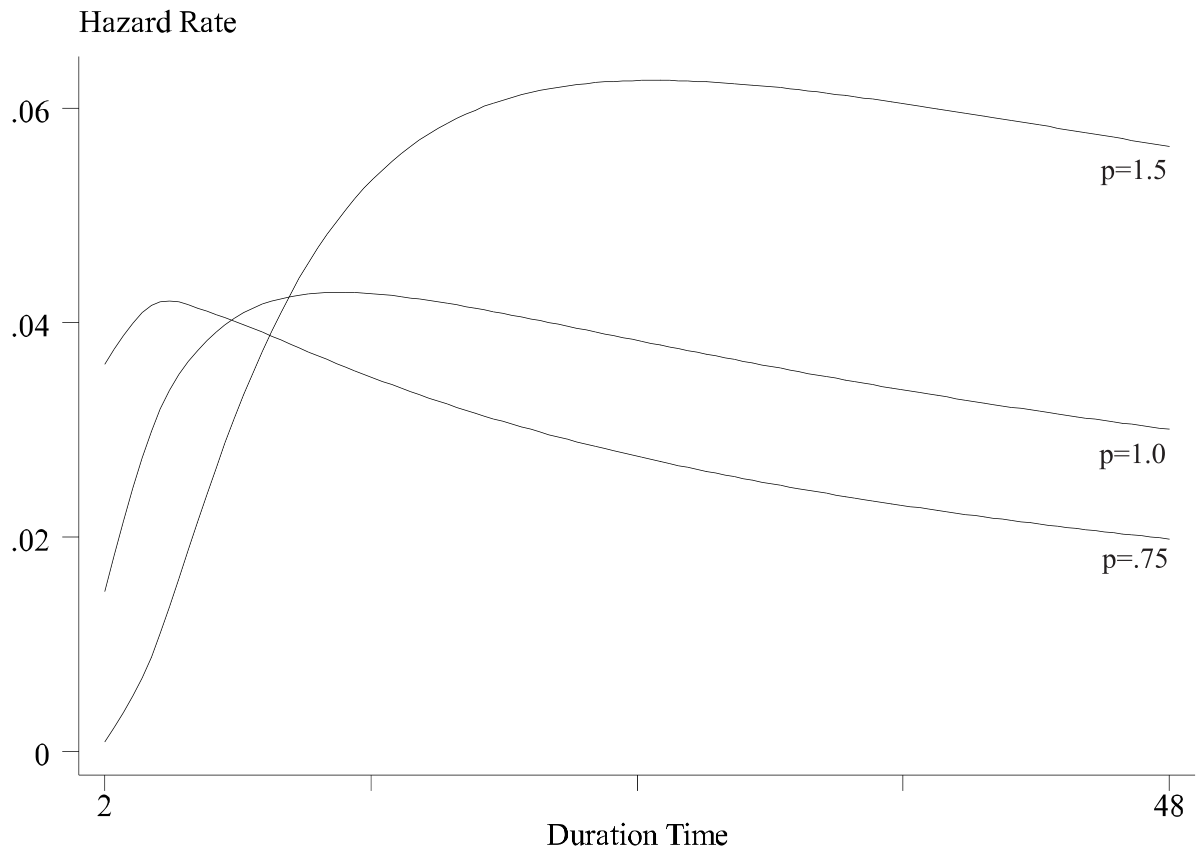

Figure 4: *This figure graphs some typically shaped hazard rates for the log-normal model.*

Gompertz

$$h(t) = \lambda \exp \gamma t, \tag{18}$$

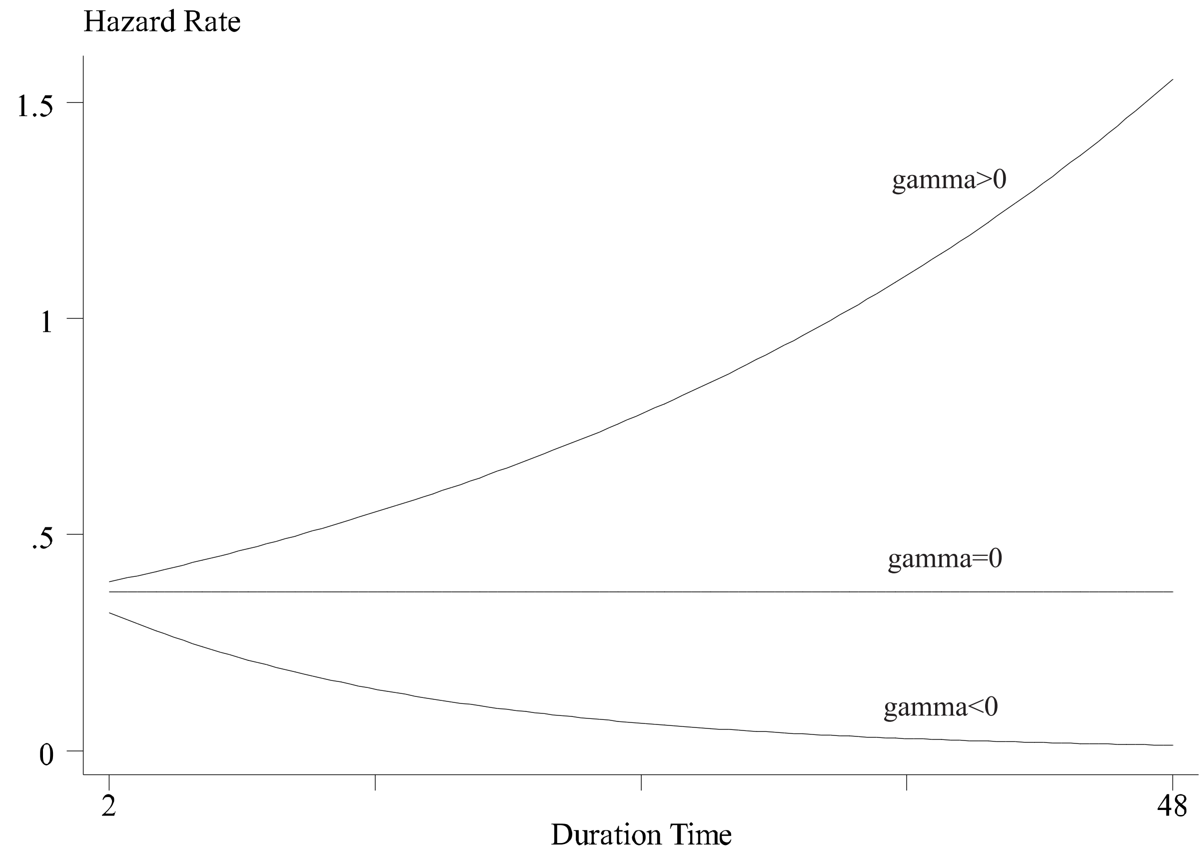

Figure 5: *This figures graphs some typically shaped hazard rates for the Gompertz model.*

$$S(t) = \exp\{-\lambda\gamma^{-1}(e^{\lambda t} - 1)\}, \tag{19}$$

Likelihood

$$\mathcal{L} = \prod_{i=1}^{n} \{f(t_i)\}^{\delta_i}\{S(t_i)\}^{1-\delta_i}, \tag{20}$$

Weibull:

$$f(t) = \lambda p(\lambda t)^{p-1} \exp^{-(\lambda t)^p},$$

$$S(t) = \exp^{-(\lambda t)^p}.$$

$$\mathcal{L} = \prod_{i=1}^{n} \{\lambda p(\lambda t)^{p-1} \exp^{-(\lambda t)^p}\}^{\delta_i}\{\exp^{-(\lambda t)^p}\}^{1-\delta_i}. \tag{21}$$

Exponential:

$$\mathcal{L} = \prod_{i=1}^{n} \{\lambda(t) \exp^{-\lambda(t)}\}^{\delta_i}\{\exp^{-\lambda(t)}\}^{1-\delta_i}, \tag{22}$$

$$\mathcal{L} = \prod_{i=1}^{n} \lambda^{\delta_i} \exp^{-\lambda(t)}. \tag{23}$$

$$L = \sum_{i=1}^{n} \delta_i \log \lambda - \lambda \sum_{i=1}^{n} t_i. \tag{24}$$

$$\frac{\partial \log L}{\partial \lambda} = \frac{\sum_{i=1}^{n} \delta_i}{\lambda} - \sum_{i=1}^{n} t_i. \tag{25}$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{n} \delta_i}{\sum_{i=1}^{n} t_i}, \tag{26}$$