

Slide Series 2: Models of Voting

Anti-Majoritarian Restrictions, Condorcet's Paradox, and Arrow's Theorem

- The Problem: Internally consistent preference orders (complete and transitive) can be, when aggregated into a social choice, intransitive: they can cycle.
- This is Condorcet's Paradox
- This *is* a real-world problem.
- And it's a problem if you care about majority voting!
- How big of a problem is it? Enter Kenneth Arrow.

Arrow's Impossibility Theorem

- Articulated in *Social Choice and Individual Values* (1951)
- In a nutshell (more precise definition later): given a set of minimal requirements for the way in which a group makes decisions, there exists NO method of aggregating individual preferences into a coherent group choice that jointly satisfies these minimal requirements.
- Importance? In very realistic group choice settings, there is no way to aggregate preferences in a way that jointly maintains the minimal requirements for group choice.
- WHAT ARE THE CONDITIONS? Shepsle and Bonchek do a nice job of discussing them. I'll adapt their notation.

Assumptions and Conditions

- Start with rationality assumption: Preferences are complete and transitive.
- Now add in 4 minimal requirements:

University Admissibility Condition

Any strong or weak preference order is permissible: there are no restrictions on how you order alternatives. Thus, if x, y , and z are the alternatives, any complete and transitive preference relation is permissible; hence *universal admissibility*.

Unanimity Condition

If $x P y \forall G$, then $x P y$ for G .

Independence of Irrelevant Alternatives (IIA) Condition

The presence of a third alternative, z , cannot affect the preference arrangement between two alternatives x and y .

Example: Suppose $H. Clinton > G. Pataki$ in a two-candidate election. Now suppose $J. McCain$ enters the race. IIA says the preference arrangement between Clinton and Pataki is unaffected by the presence of McCain. In other words, if Clinton is preferred to Pataki *without McCain* in the race, Clinton will still be preferred to Pataki *with McCain* in the race IF IIA PROPERTY HOLDS (it may not!).

No Dictator

\exists no $i \in G$ whose own preferences dictates the group's choice.

These are all minimal “low-hurdle” conditions for group decision-making. In fact, you would just “assume” that they hold even if you didn't know to articulate them.

Arrow's Theorem

- The importance of Arrow's theorem was to show under these minimal requirements for group decision-making, it is IMPOSSIBLE to jointly satisfy the four conditions just discussed.
- Arrow's Theorem (adapted definition from S&B, p. 67): **There exists no mechanism for translating the preferences of rational individuals into a coherent group preference that simultaneously satisfies the conditions of universal admissibility, unanimity, IIA, and no dictator.**
- So What? (Good question to ask)
- Here are the answers:

Arrow shows that Condorcet Winners cannot be guaranteed.

Arrow shows us that there is a trade-off between social-rationality and power: you want a clear-cut decision to be made? IMPOSE A DICTATOR ... but this violates one of the fairness conditions.

Does cycling really occur? YES.

If cycles do occur, then it is REALLY RISKY to treat groups *as if* they are individuals. Why? The social choice may be intransitive—irrational by our definition—even if the individual choosers are rational (this, of course *is* the Condorcet Paradox).

Original Intent? John Danforth R-MO (quoted in S&B taken from *Washington Post*: “Any judge who tries to make legislative history out of the free-for-all that takes place on the floor of the Senate is on very dangerous grounds ... It is a muddle.” (S&B, p. 71).

Paradox Lost?

- What is Arrow really saying?
- Suppose we relax one of the conditions?
- Which one is most realistic to relax?
- Suppose we work with the universal admissibility condition?

Toward the Median

- What does universal admissibility say?
- ANY transitive preference order is permissible?
- Does this always make sense?
- Example

Alternatives:

ϕ is the “status quo.”

L is the leftist alternative.

R is the rightist alternative.

Let’s draw a picture:

Does this make sense?

What is the leftist’s preference arrangement?

What is the rightist’s preference arrangement?

What about a centrist?

The Power of the Median

- For the three alternatives L, ϕ, R , there exists no realistic condition where ϕ is the worst outcome.
- If true, then: $L > R > \phi$ and $R > L > \phi$ is *disallowed*.
- What have we just done? WE HAVE RELAXED THE CONDITION OF UNIVERSAL ADMISSIBILITY.
- We have consensus for ϕ . That is: ϕ would be the winner?
- What's "special" about ϕ ?
- It's not "worst"!
- **Black's Single-Peakedness Theorem:** Suppose there exists a set of alternatives A . For any subset of 3 alternatives in A , if one is never worst among the three for any group member, then majority rule will yield a transitive preference.
- OK, so what?
- By relaxing universal admissibility, we may get to a transitive preference.
- We also see something else ...

Single-Peakedness

- What are your preferences over grades?
- Over cups of coffee?
- Over abortion?
- Over Iraq?
- Over big government?
- **KEY CONCEPT:** Single-Peakedness—if alternatives can be spatially represented and the utility for these alternatives has a maxima from which utility for other alternatives decreases, then preferences can be said to be **SINGLE-PEAKED**.

Example

- How many troops will you send?
- Choice Set: $[0, 100]$
- Voters: K, J, X, Y, Z
- Preferences:
 - $K \rightarrow 0$
 - $J \rightarrow 20$
 - $X \rightarrow 55$
 - $Y \rightarrow 65$
 - $Z \rightarrow 100$
- Status quo, ϕ , is 30.
- Let's draw pictures.

Winsets

- The sets overlap for voters X, Y, Z .
- If the status quo is 30, we can see that X, Y, Z have “preferred to” points that are greater than 30.
- The set of points (i.e. troops deployed) the majority prefers to the status quo is the WINSET:

$$W = P_x \cap P_y \cap P_z \tag{1}$$

It is the intersection of points.

- Question: Do any alternatives have an *empty* winset?
- YES: $W_x = \emptyset$
- So what?
- IF MEMBERS OF A GROUP HAVE SINGLE-PEAKED PREFERENCES, THEN THE IDEAL POINT OF THE MEDIAN VOTER HAS AN EMPTY WINSET.
- There exists no points that is “better” than X 's position.
- This is the essence of the **MEDIAN VOTER THEOREM**

Black's Median Voter Theorem

- The Theorem: “If members of group G have single-peaked preferences, then the ideal point of the median voter has an empty winset.”

- Implications?

→ Know the median, know the winner.

→ Predicts CONVERGENCE.

- What *is* convergence?

- Names to Know: Hotelling, Downs

- Question: Why is La Salsa and Chipotle 200 meters apart?

- What does “moderation” mean, politically?

- What about polarization?